Abschlussprüfung 1995
an den Realschulen in Bayern

Mathematik II — Aufgabengruppe B

Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 28.08.2013

Aufgabe B1

B 1.0 $p: y = 0{,}25x^2 + x - 4$ und $h: y = 0{,}5x + 2$

B 1.1 $y = 0{,}25(x^2 + 4x + 2^2 - 2^2) - 4$

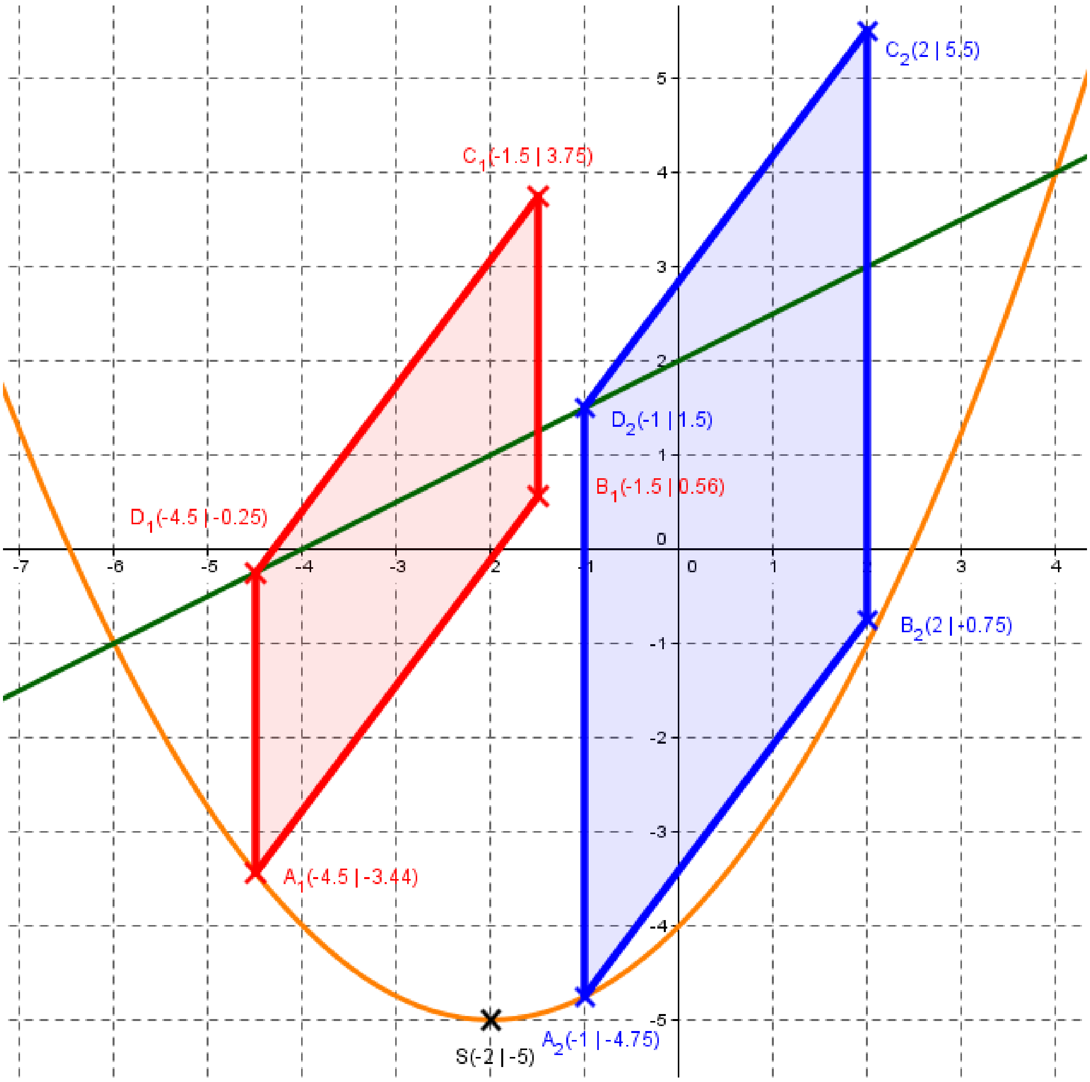
$\Leftrightarrow y = 0{,}25(x + 2)^2 - 5 \Rightarrow S(-2 \mid -5)$

x	-7,0	-6,0	-5,0	-4,0	-3,0	-2,0	-1,0	0,0	1,0	2,0	3,0	4,0	5,0
y	1,25	-1,00	-2,75	-4,00	-4,75	-5,00	-4,75	-4,00	-2,75	-1,00	1,25	4,00	7,25

B 1.2

$A_1(-4{,}5 \mid -3{,}4375)$ $B_1(-1{,}5 \mid 0{,}5625)$ $C_1(-1{,}5 \mid 3{,}75)$ $D_1(-4{,}5 \mid -0{,}25)$

$A_2(-1 \mid -4{,}75)$ $B_2(0 \mid -0{,}75)$ $C_2(2 \mid 5{,}5)$ $D_2(-1 \mid 1{,}5)$



B 1.3

$$\overrightarrow{A_nD_n} = \begin{pmatrix} x - x \\ 0{,}5x + 2 - (0{,}25x^2 + x - 4) \end{pmatrix} = \begin{pmatrix} 0 \\ -0{,}25x^2 - 0{,}5x + 6 \end{pmatrix}$$

$$\overrightarrow{D_nC_n} = \overrightarrow{A_nB_n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$A(x) = \begin{vmatrix} 3 & 0 \\ 4 & -0{,}25x^2 - 0{,}5x + 6 \end{vmatrix} \text{ FE} = (-0{,}75x^2 - 1{,}5x + 18) \text{ FE}$$

B 1.4 $A(x) = -0{,}75x^2 - 1{,}5x + 18$

$\Leftrightarrow \quad A(x) = -0{,}75(x^2 + 2x + 1^2 - 1^2) + 18$

$\Leftrightarrow \quad A(x) = -0{,}75(x + 1)^2 + 18{,}75$

Damit ist der maximale Flächeninhalt $A_{max} = 18{,}75$ FE für $x = -1$

B 1.5

$\overline{A_nB_n}(x) = \sqrt{[3]^2 + [4]^2}$ LE $= 5$ LE

$\overline{A_nD_n}(x) = \sqrt{[0]^2 + [-0{,}25x^2 - 0{,}5x + 6]^2}$ LE $= -0{,}25x^2 - 0{,}5x + 6$ LE

Also: $-0{,}25x^2 - 0{,}5x + 6 = 5$

$\Leftrightarrow \quad -0{,}25x^2 - 0{,}5x + 1 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0{,}5 \pm \sqrt{(-0{,}5)^2 - 4 \cdot (-0{,}25) \cdot 1}}{-0{,}5}$$

$$= \frac{0{,}5 \pm \sqrt{1{,}25}}{-0{,}5} \Rightarrow x_1 = -3{,}24 \text{ und } x_2 = 1{,}24 \qquad \mathbb{L} = \{-3{,}24;\ 1{,}24\}$$

[Kontrolle]

$A_3(1{,}24|-2{,}38)$ $B_3(3{,}24 \mid 1{,}62)$ $C_3(2 \mid 5{,}5)$ $D_3(1{,}24 \mid 2{,}62)$

$A_4(-3{,}24|-4{,}62)$ $B_4(-0{,}24|-0{,}62)$ $C_4(-0{,}24|4{,}38)$ $D_4(-3{,}24|0{,}38)$

B 1.6

Wenn B_5 und B_6 auf der x-Achse liegt, dann hat A_5 bzw. A_6 den y-Wert -4.

A_5: $\mathbf{-4 = 0{,}25x^2 + x - 4}$

$\Leftrightarrow \quad 0{,}25x^2 + x = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4 \cdot (0{,}25) \cdot 0}}{0{,}5}$$

$$= \frac{-1 \pm \sqrt{1}}{0{,}5} \Rightarrow x_1 = 0 \text{ und } x_2 = -4 \qquad \mathbb{L} = \{-4;\ 0\}$$

Damit $A_5(-4 \mid -4)$ und $A_6(0 \mid -4)$.

Aufgabe B2

B 2.1 und B 2.2

$\overline{AP}$: Dreieck ABP

$\sphericalangle APB = 180° - 32° - 65° = 83°$

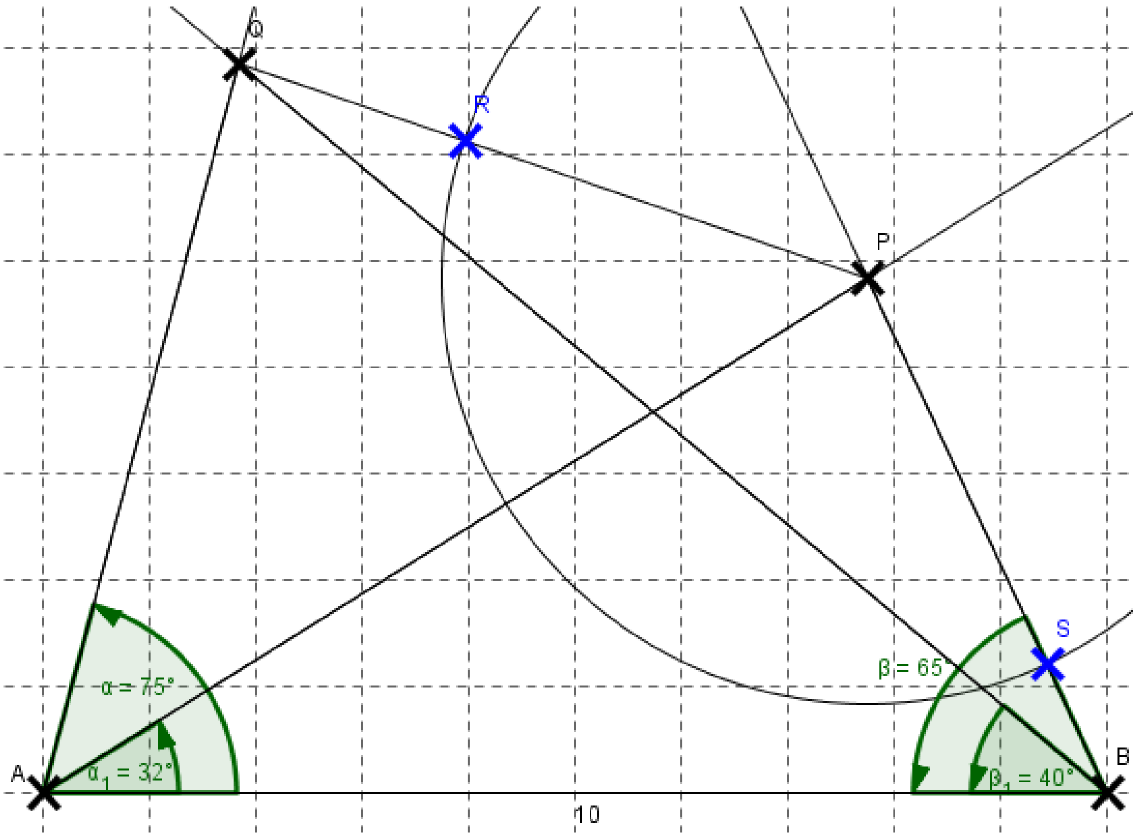
$$\frac{\overline{AP}}{\sin\beta} = \frac{\overline{AB}}{\sin\sphericalangle APB} \Leftrightarrow \overline{AP} = \frac{\overline{AB}\cdot\sin\beta}{\sin\sphericalangle APB} = \frac{100\text{ m}\cdot\sin 65°}{\sin 83°} = 91{,}3\text{ m}$$

$\overline{AQ}$: Dreieck ABQ

$\sphericalangle AQB = 180° - 75° - 40° = 65°$

$$\frac{\overline{AQ}}{\sin\beta_1} = \frac{\overline{AB}}{\sin\sphericalangle AQB} \Leftrightarrow \overline{AQ} = \frac{\overline{AB}\cdot\sin\beta_1}{\sin\sphericalangle AQB} = \frac{100\text{ m}\cdot\sin 40°}{\sin 65°} = 70{,}9\text{ m}$$

B 2.3



B 2.4

Dreieck ABQ:

$$\frac{\overline{BQ}}{\sin\alpha} = \frac{\overline{AB}}{\sin\sphericalangle AQB} \Leftrightarrow \overline{BQ} = \frac{\overline{AB}\cdot\sin\alpha}{\sin\sphericalangle AQB} = \frac{100\text{ m}\cdot\sin 75°}{\sin 65°} = 106{,}6\text{ m}$$

Dreieck ABP:

$$\frac{\overline{BP}}{\sin\alpha_1} = \frac{\overline{AP}}{\sin\beta} \Leftrightarrow \overline{BP} = \frac{\overline{AP}\cdot\sin\alpha_1}{\sin\beta} = \frac{91{,}3\text{ m}\cdot\sin 32°}{\sin 65°} = 53{,}4\text{ m}$$

Dreieck BPQ:

$$\overline{QP}^2 = \overline{BQ}^2 + \overline{BP}^2 - 2 \cdot \overline{BQ} \cdot \overline{BP} \cos \beta_2$$

$$\Leftrightarrow \overline{QP}^2 = (106,6^2 + 53,4^2 - 2 \cdot 106,6 \cdot 53,4 \cdot \cos 25°) m^2 = 3896,9\ m^2$$

$$\Leftrightarrow \overline{QP} = 62,4\ m$$

$$\frac{\overline{BQ}}{\sin \sphericalangle QPB} = \frac{\overline{QP}}{\sin \beta_2} \Leftrightarrow \sin \sphericalangle QPB = \frac{\overline{BQ} \cdot \sin \beta_2}{\overline{QP}}$$

$$\Leftrightarrow \sin \sphericalangle QPB = \frac{106,6\ m \cdot \sin 25°}{62,4\ m} = 0,72$$

$$\Leftrightarrow \sphericalangle QPB = (46,2°) \text{ und } 180° - 46,22° = 133,8°$$

$$A_{Kreisstückerl} = r^2 \cdot \pi \cdot \frac{133,8°}{360°} = 40^2 \cdot \pi \cdot \frac{133,8°}{360°} m^2 = 1868,2\ m^2$$

$$A_{ABP} = 0,5 \cdot \overline{AB} \cdot \overline{AP} \cdot \sin \alpha_1 = 0,5 \cdot 100 \cdot 91,3 \cdot \sin 32°\ m^2 = 2419,1\ m^2$$

$$A_{APQ} = 0,5 \cdot \overline{AQ} \cdot \overline{AP} \cdot \sin \alpha_2 = 0,5 \cdot 70,9 \cdot 91,3 \cdot \sin 43°\ m^2 = 2207,3\ m^2$$

$$A_{Park} = 2419,1\ m^2 + 2207,3\ m^2 - 1868,2\ m^2 = 2758,20\ m^2$$

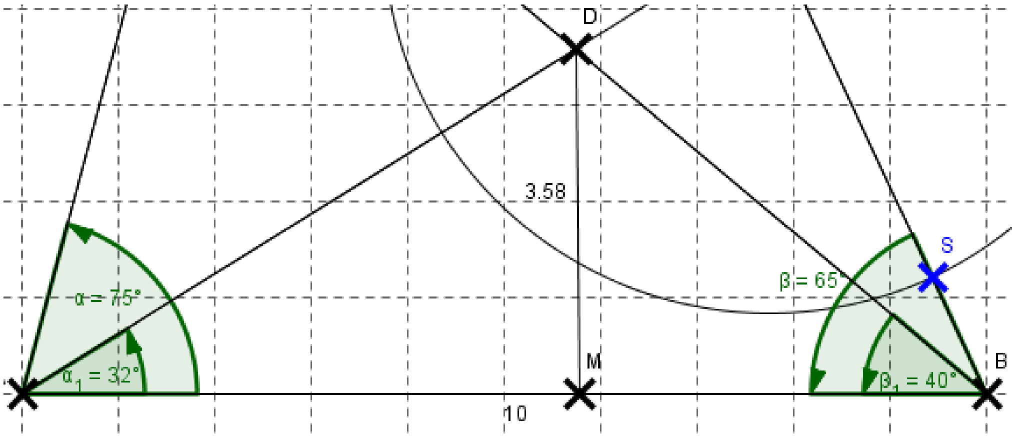
B 2.5 Dreieck ABD

$$\frac{\overline{AD}}{\sin \beta_1} = \frac{\overline{AB}}{\sin \sphericalangle AEB} \Leftrightarrow \overline{AD} = \frac{\overline{AB} \cdot \sin \beta_1}{\sin \sphericalangle AEB} = \frac{100\ m \cdot \sin 40°}{\sin 108°} = 67,6\ m$$

$$A_{ABD} = 0,5 \cdot \overline{AB} \cdot \overline{AD} \cdot \sin \alpha_1 = 0,5 \cdot 100 \cdot 67,6 \cdot \sin 32°\ m^2 = 1791,1\ m^2$$

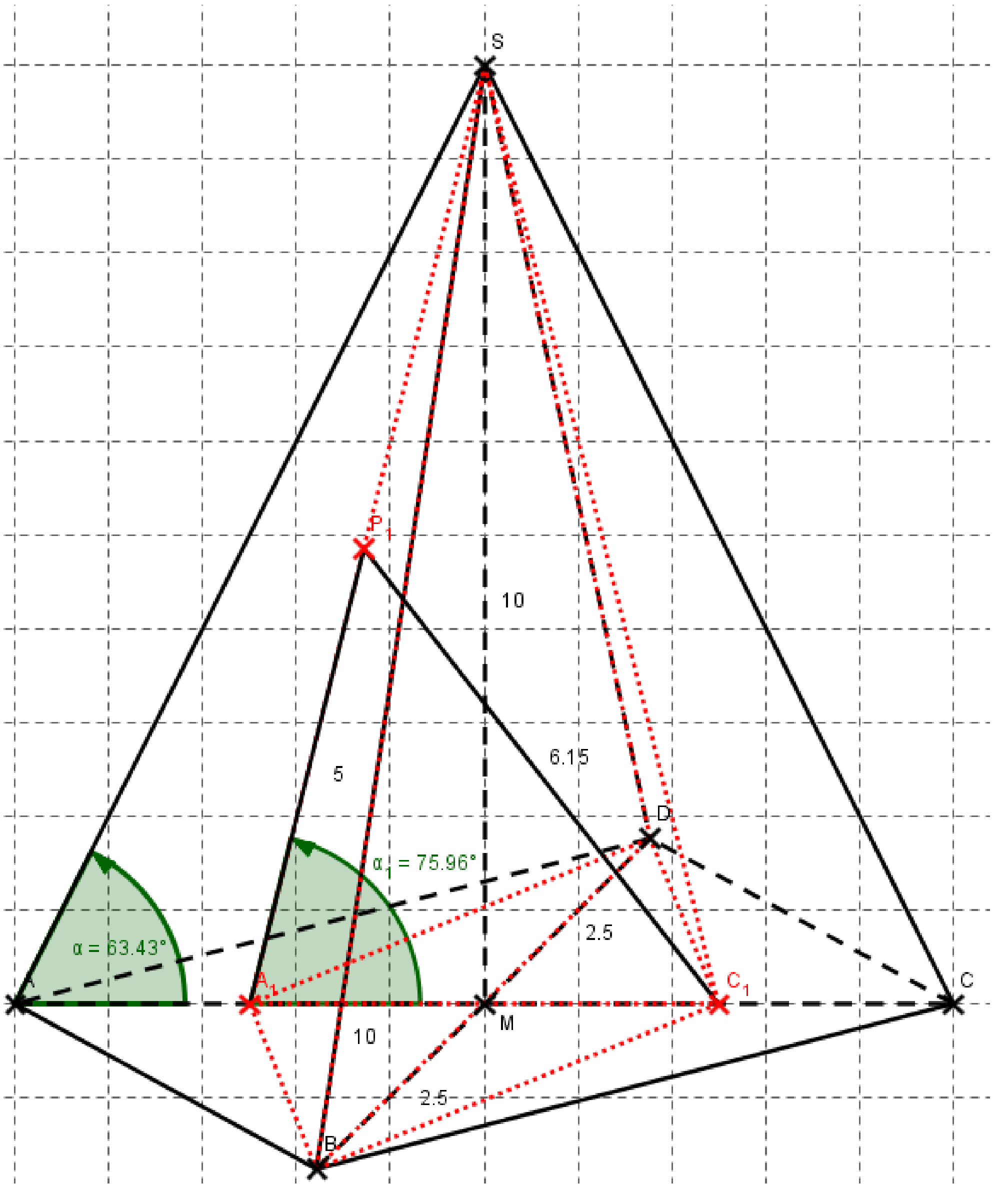
$$A_{ABD} = 0,5 \cdot \overline{AB} \cdot h = 1791,1\ m^2$$

$$\Leftrightarrow 1791,1\ m^2 = 0,5 \cdot 100\ m \cdot h \quad \Leftrightarrow \quad h = 35,8\ m$$



Aufgabe B3

B 3.1



B 3.2

$$\tan\alpha = \frac{\overline{MS}}{\overline{AM}} = \frac{10\ cm}{5\ cm} = 2 \quad \Leftrightarrow \alpha = 63{,}43°$$

$$\overline{AS}^2 = \overline{MS} \cdot \overline{AM} = (10\ cm)^2 \cdot (5\ cm)^2 = 125\ cm^2$$

$$\Leftrightarrow \overline{AS} = 11{,}18\ cm$$

B 3.3

$$\overline{A_1M} = 5\ cm - 2{,}5\ cm = 2{,}5\ cm$$

$$\overline{A_1B}^2 = \overline{A_1M}^2 + \overline{MB}^2$$

$$\Leftrightarrow \overline{A_1B}^2 = (2{,}5\ cm)^2 + (5\ cm)^2 = 31{,}25\ cm^2$$

$$\Leftrightarrow \overline{A_1B} = \overline{BC_1} = 5{,}59\ cm$$

$$\overline{A_1S}^2 = \overline{A_1M}^2 + \overline{MS}^2$$

$$\Leftrightarrow \overline{A_1S}^2 = (2{,}5\ \text{cm})^2 + (10\ \text{cm})^2 = 106{,}25\ \text{cm}^2$$

$$\Leftrightarrow \overline{A_1S} = \overline{C_1S} = 10{,}31\ \text{cm}$$

$$\overline{BS} = \overline{AS} = 11{,}18\ \text{cm}$$

$$\overline{A_1B}^2 = \overline{A_1S}^2 + \overline{BS}^2 - 2 \cdot \overline{A_1S} \cdot \overline{BS} \cdot \cos \sphericalangle A_1SB$$

$$\Leftrightarrow \cos \sphericalangle A_1SB = \frac{\overline{A_1B}^2 - \overline{A_1S}^2 - \overline{BS}^2}{-2 \cdot \overline{A_1S} \cdot \overline{BS}}$$

$$\Leftrightarrow \cos \sphericalangle A_1SB = \frac{31{,}25\ \text{cm}^2 - 106{,}25\ \text{cm}^2 - (11{,}18\ \text{cm})^2}{-2 \cdot 10{,}31\ \text{cm} \cdot 11{,}18\ \text{cm}}$$

$$\Leftrightarrow \cos \sphericalangle A_1SB = 0{,}87 \quad \Leftrightarrow \sphericalangle A_1SB = 29{,}8°$$

$$A_{Boden} = 0{,}5 \cdot \overline{A_1C_1} \cdot \overline{BD} = 0{,}5 \cdot 5\ \text{cm} \cdot 10\ \text{cm} = 25\ \text{cm}^2$$

$$4 \cdot A_{Dreieck} = 4 \cdot 0{,}5 \cdot \overline{A_1S} \cdot \overline{BS} \cdot \sin \sphericalangle A_1SB$$

$$\Leftrightarrow 4 \cdot A_{Dreieck} = 2 \cdot 10{,}31\ \text{cm} \cdot 11{,}18\ \text{cm} \cdot \sin 29{,}8° = 114{,}57\ \text{cm}^2$$

$$O = 25\ \text{cm}^2 + 114{,}57\ \text{cm}^2 = 139{,}57\ \text{cm}^2$$

B 3.4

$\sphericalangle C_1A_1P_1$:

$$\tan \alpha_1 = \frac{\overline{MS}}{\overline{A_1M}} = \frac{10\ \text{cm}}{2{,}5\ \text{cm}} = 4$$

$$\Leftrightarrow \alpha_1 = 75{,}96°$$

$$\overline{P_1C_1}^2 = \overline{A_1C_1}^2 + \overline{A_1P_1}^2 - 2 \cdot \overline{A_1C_1} \cdot \overline{A_1P_1} \cdot \cos \alpha_1$$

$$\Leftrightarrow \overline{P_1C_1}^2 = (5\ \text{cm})^2 + (5\ \text{cm})^2 - 2 \cdot 5\text{cm} \cdot 5\ \text{cm} \cdot \cos 75{,}96°$$

$$\Leftrightarrow \overline{P_1C_1}^2 = 37{,}87\ \text{cm}^2$$

$$\Leftrightarrow \overline{P_1C_1} = 6{,}15\ \text{cm}$$

B 3.5

$\sphericalangle A_2SC_2 = 40° \Rightarrow \sphericalangle C_2A_2S = 70°$ und $\sphericalangle SC_2A_2 = 70°$ sowie $\sphericalangle A_2SM = 20°$

$$\frac{\overline{MS}}{\sin \sphericalangle C_2A_2S} = \frac{\overline{A_1M}}{\sin \sphericalangle A_2SM}$$

$$\Leftrightarrow \overline{A_1M} = \frac{\overline{MS} \cdot \sin \sphericalangle A_2SM}{\sin \sphericalangle C_2A_2S} = \frac{10\ \text{cm} \cdot \sin 20°}{\sin 70°} = 3{,}64\ \text{cm}$$

Somit ist x = 5 cm – 3,64 cm = 1,36