Abschlussprüfung 1994
an den Realschulen in Bayern

Mathematik II — Aufgabengruppe B

Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 05.08.2013

Aufgabe B1 **p: $y = 0{,}25x^2 + 0{,}5x + 2{,}25$** **g: $y = x - 3$**

B 1.1 $y = 0{,}25(x^2 + 2x + 1^2 - 1^2) + 2{,}25$

$\Leftrightarrow y = 0{,}25(x + 1)^2 + 2 \quad \Rightarrow S(-1 \mid 2)$

x	-2,0	-1,0	0,0	1,0	2,0	3,0	4,0
y	2,25	2,00	2,25	3,00	4,25	6,00	8,25

B 1.2

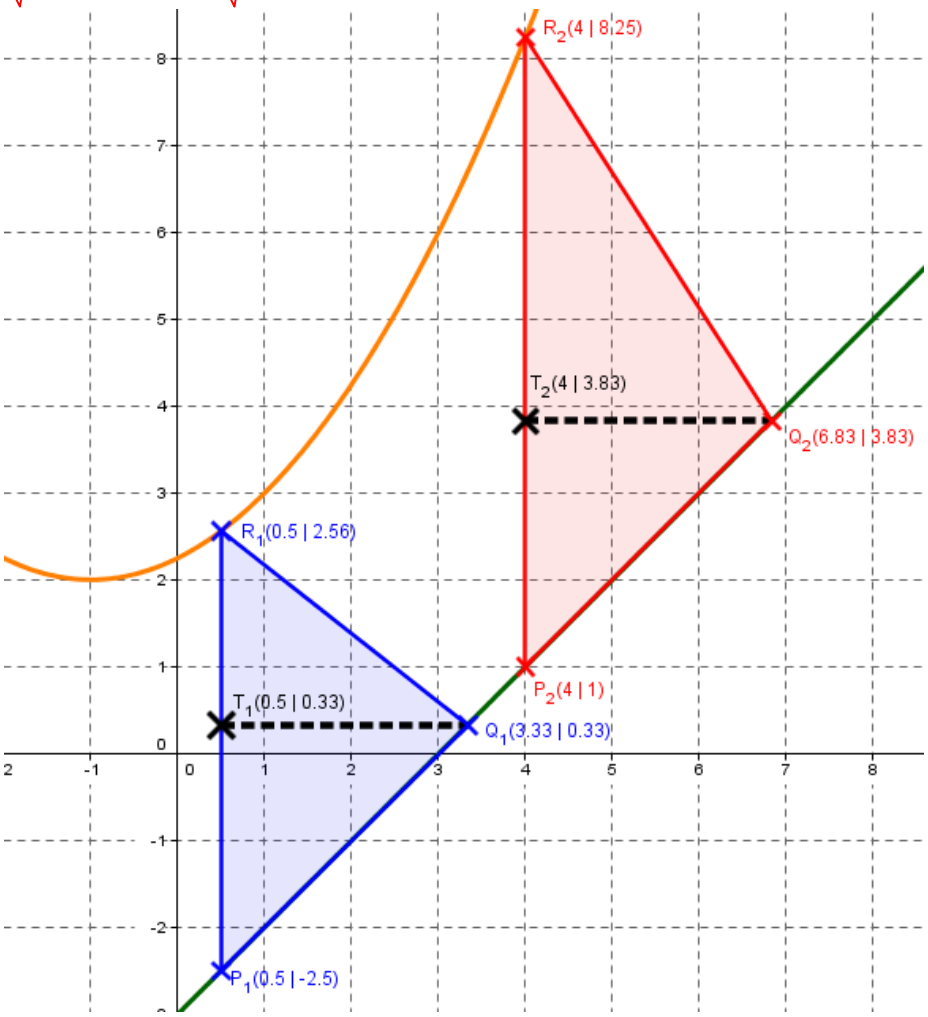
$R_1(0{,}5 \mid 2{,}5625)$, $P_1(0{,}5 \mid -2{,}5)$

$\overline{P_nQ_n} = \sqrt{x^2 + x^2} \quad \Leftrightarrow 4 \text{ LE} = \sqrt{2}x \quad \Leftrightarrow x = \frac{4}{\sqrt{2}}$ LE $x = y$, da die Gerade die Steigung 1 hat.

$Q_1(0{,}5 + \frac{4}{\sqrt{2}} \mid -2{,}5 + \frac{4}{\sqrt{2}})$ also $Q_1(3{,}33 \mid 0{,}33)$

$R_2(4 \mid 8{,}25)$, $P_2(4 \mid 1)$

$Q_2(4 + \frac{4}{\sqrt{2}} \mid 1 + \frac{4}{\sqrt{2}})$ also $Q_2(6{,}83 \mid 3{,}83)$



B 1.3

Aus 1.2: $\sqrt{x^2 + x^2}$

$\overline{T_nQ_n} = \frac{4}{\sqrt{2}}$ LE $= \frac{2 \cdot \sqrt{2} \cdot \sqrt{2}}{\sqrt{2}}$ LE $= 2\sqrt{2}$ LE

B 1.4

$\overline{P_nR_n} = \sqrt{0^2 + [0{,}25x^2 + 0{,}5x + 2{,}25 - (x - 3)]^2}$ LE

$\Leftrightarrow \overline{P_nR_n} = (0{,}25x^2 - 0{,}5x + 5{,}25)$ LE

$A(x) = 0{,}5 \cdot \overline{T_nQ_n} \cdot \overline{P_nR_n}$

$\Leftrightarrow A(x) = 0{,}5 \cdot 2\sqrt{2} \cdot (0{,}25x^2 - 0{,}5x + 5{,}25)$ FE

$\Leftrightarrow A(x) = \sqrt{2} \cdot (0{,}25x^2 - 0{,}5x + 5{,}25)$ FE

B 1.5

$A(x) = \sqrt{2} \cdot (0{,}25x^2 - 0{,}5x + 5{,}25)$

$\Leftrightarrow A(x) = \sqrt{2} \cdot [0{,}25(x^2 - 2x + 1^2 - 1^2) + 5{,}25]$

$\Leftrightarrow A(x) = \sqrt{2} \cdot [0{,}25(x - 1)^2 + 5]$

$\Leftrightarrow A(x) = \sqrt{2} \cdot 0{,}25(x - 1)^2 + 7{,}07$

Also ist $A_{min} = 7{,}07$ FE für $x = 1$.

$P_0(1 \mid -2)$

B 1.6

$\overline{Q_nR_n} = \overline{P_nQ_n} = 4$ LE

Winkel an der Basis: $90° : 2 = 45°$

$\sin 45° = \frac{\overline{P_nR_n}}{\overline{P_nQ_n}} = \frac{4}{0{,}25x^2 - 0{,}5x + 5{,}25}$

$\Leftrightarrow 5{,}67 = 0{,}25x^2 - 0{,}5x + 5{,}25$

$\Leftrightarrow 0{,}25x^2 - 0{,}5x - 0{,}42 = 0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0{,}5 \pm \sqrt{(-0{,}5)^2 - 4 \cdot 0{,}25 \cdot (-0{,}42)}}{0{,}5}$

$= \frac{0{,}5 \pm \sqrt{0{,}67}}{0{,}5} \Rightarrow x_1 = 2{,}64$ und $x_2 = -0{,}64$ $\quad \mathbb{L} = \{2{,}64; -0{,}64\}$

$P_3(2{,}64 \mid -0{,}63)$ und $P_4(-0{,}64 \mid -3{,}64)$

Aufgabe B2

B 2.1

$\overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2$

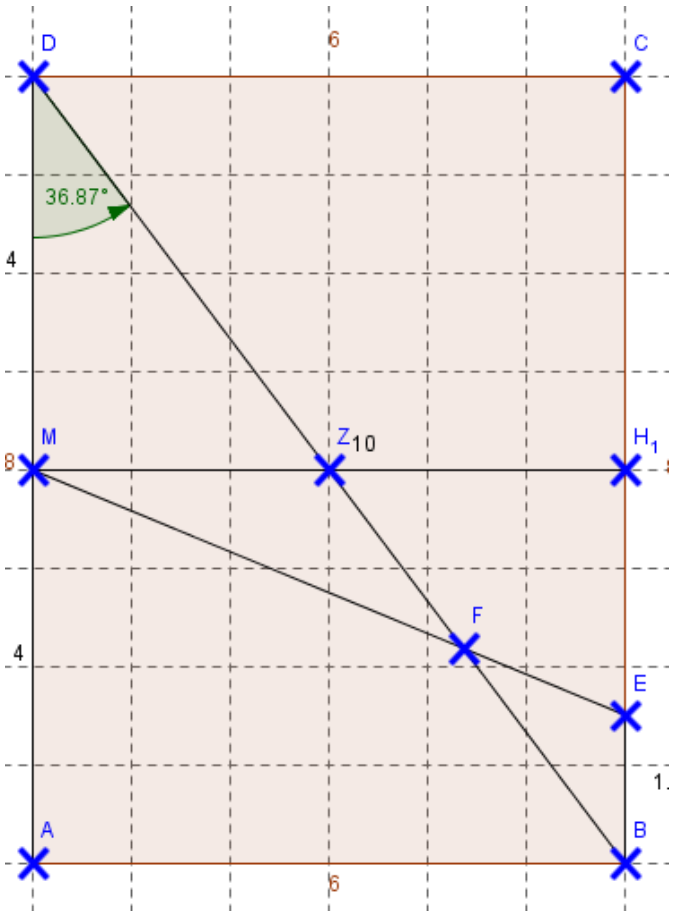
$\Leftrightarrow \overline{BD}^2 = (6\ cm)^2 + (8\ cm)^2$

$\Leftrightarrow \overline{BD}^2 = 100\ cm^2$

$\Leftrightarrow \overline{BD} = 10\ cm$ und $\overline{DZ} = 5\ cm$

$$\tan \sphericalangle ADB = \frac{\overline{AB}}{\overline{AD}} = \frac{6\ cm}{8\ cm} = 0{,}75$$

$\Leftrightarrow \sphericalangle ADB = 36{,}87°$



B 2.2

$\overline{MH_1} = 6\ cm$; $\overline{EH_1} = 2{,}5\ cm$;

$\overline{MZ} = 3\ cm$; $\overline{DM} = 4\ cm$

$$\tan \sphericalangle EMH_1 = \frac{\overline{EH_1}}{\overline{MH_1}} = \frac{2{,}5\ cm}{6\ cm} = 0{,}42$$

$\Leftrightarrow \sphericalangle EMH_1 = 22{,}62°$

$\sphericalangle FMD = 90° + 22{,}62° = 112{,}62°$

$\sphericalangle DFM = 180° - 112{,}62° - 36{,}87° = 30{,}51°$

$$\frac{\overline{DF}}{\sin \sphericalangle FMD} = \frac{\overline{DM}}{\sin \sphericalangle DFM} \Leftrightarrow \overline{DF} = \frac{\overline{DM} \cdot \sin \sphericalangle FMD}{\sin \sphericalangle DFM}$$

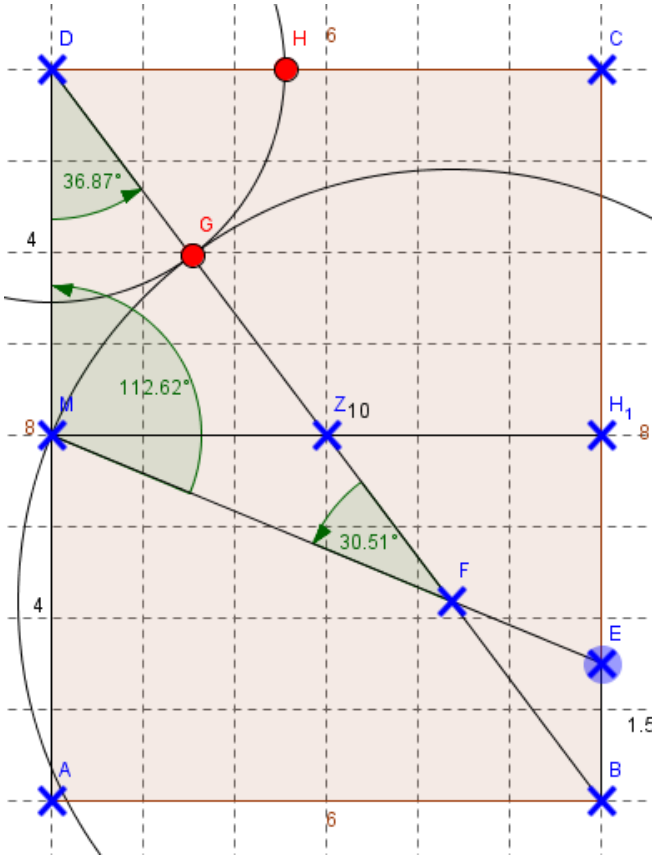
$$\Leftrightarrow \overline{DF} = \frac{4\ cm \cdot \sin 112{,}62°}{\sin 30{,}51°} = 7{,}27\ cm$$

B 2.3

$$\frac{\overline{MF}}{\sin \sphericalangle MDF} = \frac{\overline{DF}}{\sin \sphericalangle FMD} \Leftrightarrow \overline{MF} = \frac{\overline{DF} \cdot \sin \sphericalangle MDF}{\sin \sphericalangle FMD}$$

$$\Leftrightarrow \overline{MF} = \frac{7{,}27\ cm \cdot \sin 36{,}87°}{\sin 112{,}62°} = 4{,}73\ cm$$

$\sphericalangle DFM = 180° - 112{,}62° - 36{,}87° = 30{,}51°$ siehe 2.2



B 2.4
B 2.5
Dreieck DMZ:

$$A = 0{,}5 \cdot \overline{DM} \cdot \overline{MZ}$$

$$\Leftrightarrow A = 0{,}5 \cdot 3\ cm \cdot 4\ cm = 6\ cm^2$$

$$A_{Kreissektor} = r^2 \cdot \pi \cdot \frac{\sphericalangle DZM}{360°}$$

$$\Leftrightarrow A = 3^2 \cdot \pi \cdot \frac{180° - 36{,}87° - 90°}{360°}\ cm^2$$

$$\Leftrightarrow A = 9 \cdot \pi \cdot \frac{53{,}13°}{360°} = 4{,}17\ cm^2$$

$$A_{gesamt} = 6\ cm^2 - 4{,}17\ cm^2 = 1{,}83\ cm^2$$

Fehlender Kreissektor:

$$\sphericalangle GDH = 90° - 36{,}87° = 53{,}13°$$

$$A_{Kreissektor} = r^2 \cdot \pi \cdot \frac{\sphericalangle GDH}{360°}$$

$$\Leftrightarrow A = (\overline{DZ} - \overline{MZ})^2 \cdot \pi \cdot \frac{53{,}13°}{360°}\ cm^2$$

$$\Leftrightarrow A = (2\ cm)^2 \cdot \pi \cdot \frac{53{,}13°}{360°}\ cm^2 = 1{,}85\ cm^2$$

$$A_{Lösung} = 1{,}83\ cm^2 + 1{,}85\ cm^2 = 3{,}68\ cm^2$$

Aufgabe B3

B 3.1

$$\overline{BE}^2 = \overline{AE}^2 + \overline{AB}^2$$

$$\Leftrightarrow \overline{BE}^2 = (5\,cm)^2 + (8\,cm)^2$$

$$\Leftrightarrow \overline{BE}^2 = 89\ cm^2$$

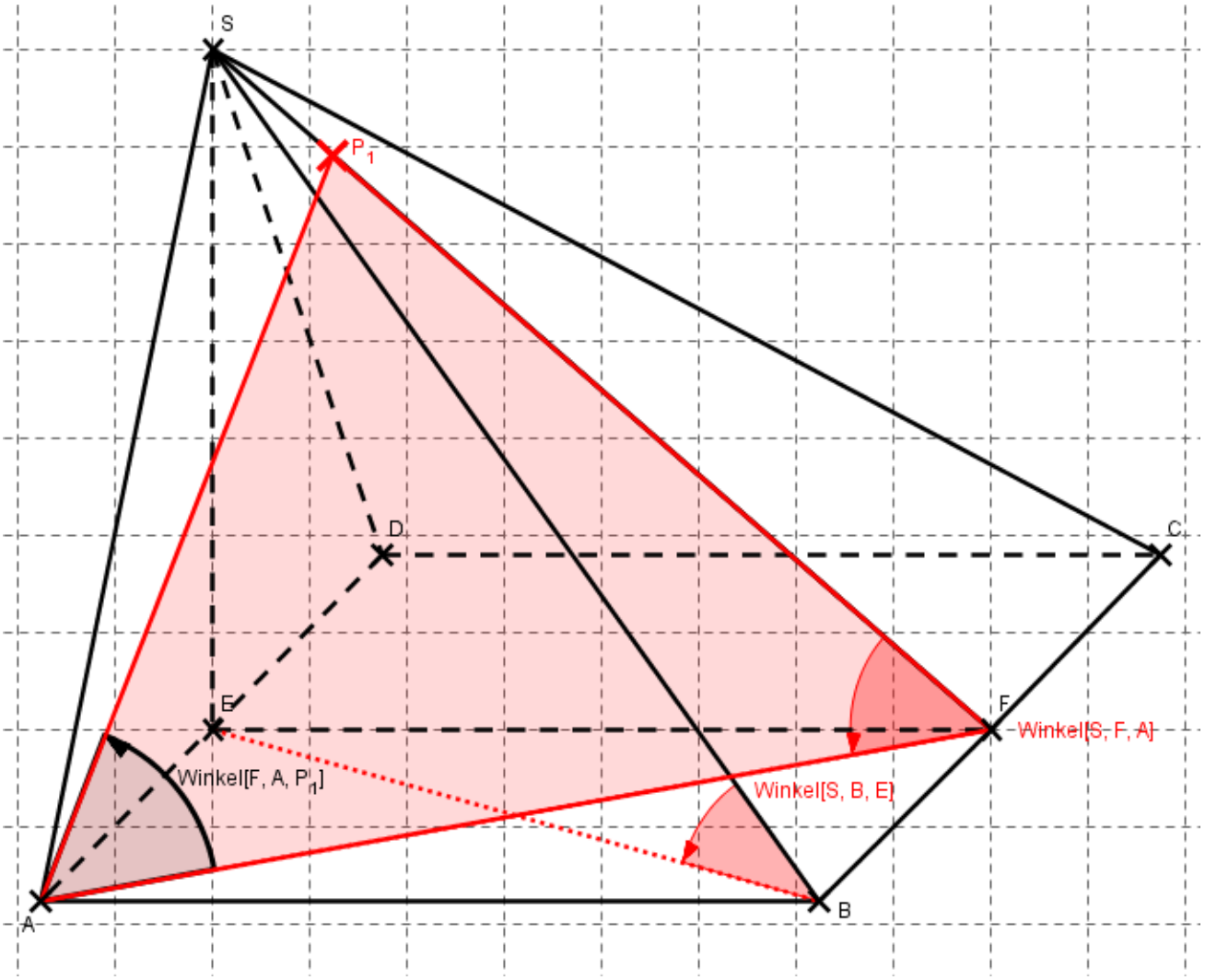
$$\Leftrightarrow \overline{BE} = 9{,}43\ cm$$

$$\tan \sphericalangle SBE = \frac{\overline{SE}}{\overline{BE}}$$

$$\Leftrightarrow \tan \sphericalangle SBE = \frac{7\ cm}{9{,}43\ cm}$$

$$\Leftrightarrow \tan \sphericalangle SBE = 0{,}74$$

$$\Leftrightarrow \sphericalangle SBE = 36{,}59°$$



B 3.2

$\overline{AF} = \overline{BE} = 9{,}43$ cm

$\overline{SF}^2 = \overline{EF}^2 + \overline{SE}^2$ $\qquad$ $\overline{AS}^2 = \overline{AE}^2 + \overline{SE}^2$

$\Leftrightarrow \overline{SF}^2 = (8\text{ cm})^2 + (7\text{ cm})^2$ $\qquad$ $\Leftrightarrow \overline{AS}^2 = (5\text{ cm})^2 + (7\text{ cm})^2$

$\Leftrightarrow \overline{SF}^2 = 113\text{ cm}^2$ $\qquad$ $\Leftrightarrow \overline{AS}^2 = 74\text{ cm}^2$

$\Leftrightarrow \overline{SF} = 10{,}63$ cm $\qquad$ $\Leftrightarrow \overline{AS} = 8{,}6$ cm

$$\overline{AS}^2 = \overline{AF}^2 + \overline{SF}^2 - 2\cdot\overline{AF}\cdot\overline{SF}\cdot\cos\sphericalangle SFA$$

$$\Leftrightarrow \cos\sphericalangle SFA = \frac{\overline{AS}^2 - \overline{AF}^2 - \overline{SF}^2}{-2\cdot\overline{AF}\cdot\overline{SF}} = \frac{8{,}6^2 - 9{,}43^2 - 10{,}63^2}{-2\cdot 9{,}43\cdot 10{,}63} = 0{,}64$$

$$\Leftrightarrow \sphericalangle SFA = 50{,}34°$$

B 3.3

$$\overline{AP_1}^2 = \overline{AF}^2 + \overline{FP_1}^2 - 2\cdot\overline{AF}\cdot\overline{FP_1}\cdot\cos\sphericalangle SFA$$

$$\Leftrightarrow \overline{AP_1}^2 = (9{,}43^2 + 9^2 - 2\cdot 9{,}43\cdot 9\cdot\cos 50{,}34°)\text{cm}^2 = 61{,}59\text{ cm}^2$$

$$\Leftrightarrow \overline{AP_1} = 7{,}85\text{ cm}$$

$$\frac{\overline{AF}}{\sin\sphericalangle FAP_1} = \frac{\overline{AP_1}}{\sin\sphericalangle SFA} \Leftrightarrow \sin\sphericalangle FAP_1 = \frac{\overline{AF}\cdot\sin\sphericalangle SFA}{\overline{AP_1}}$$

$$\Leftrightarrow \sin\sphericalangle FAP_1 = \frac{9{,}43\text{ cm}\cdot\sin 50{,}34°}{7{,}85\text{ cm}} = 0{,}92$$

$\Leftrightarrow \sphericalangle FAP_1 = 67{,}64°$ (112,36° wegen Zeichnung nicht möglich)

$A_{AFP1} = 0{,}5\cdot\overline{AF}\cdot\overline{FP_1}\cdot\sin\sphericalangle SFA = 0{,}5\cdot d_1\cdot\overline{AF}$

$\Leftrightarrow \overline{FP_1}\cdot\sin\sphericalangle SFA = d_1$

$\Leftrightarrow d_1 = 9\text{ cm}\cdot\sin 50{,}34° = 6{,}93$ cm

B 3.4

$$\overline{AP_{23}}^2 = \overline{AF}^2 + \overline{FP_{23}}^2 - 2\cdot\overline{AF}\cdot\overline{FP_{23}}\cdot\cos\sphericalangle SFA$$

$\Leftrightarrow 8^2 = 9{,}43^2 + x^2 - 2\cdot 9{,}43x\cdot\cos 50{,}34°$

$\Leftrightarrow 64 = 88{,}92 + x^2 - 12{,}04x$

$\Leftrightarrow x^2 - 12{,}04x + 24{,}92 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{12{,}04 \pm \sqrt{(-12{,}04)^2 - 4\cdot 1\cdot 24{,}92}}{2}$$

$$= \frac{12{,}04 \pm \sqrt{45{,}28}}{2} \Rightarrow x_1 = 9{,}39 \text{ und } x_2 = 2{,}66 \quad \mathbb{L} = \{2{,}66;\ 9{,}39\}$$