Abschlussprüfung 1993
an den Realschulen in Bayern

Mathematik II — **Aufgabengruppe B**

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Aufgabe B1 p_1: $y = x^2 - 4x + 1$ p_2: $y = 0{,}25x^2 - x + 4$

B 1.1 $y = x^2 - 2x + 2^2 - 2^2 + 1$

$\Leftrightarrow y = (x - 2)^2 - 3 \quad \Rightarrow S_1(2 \mid -3)$

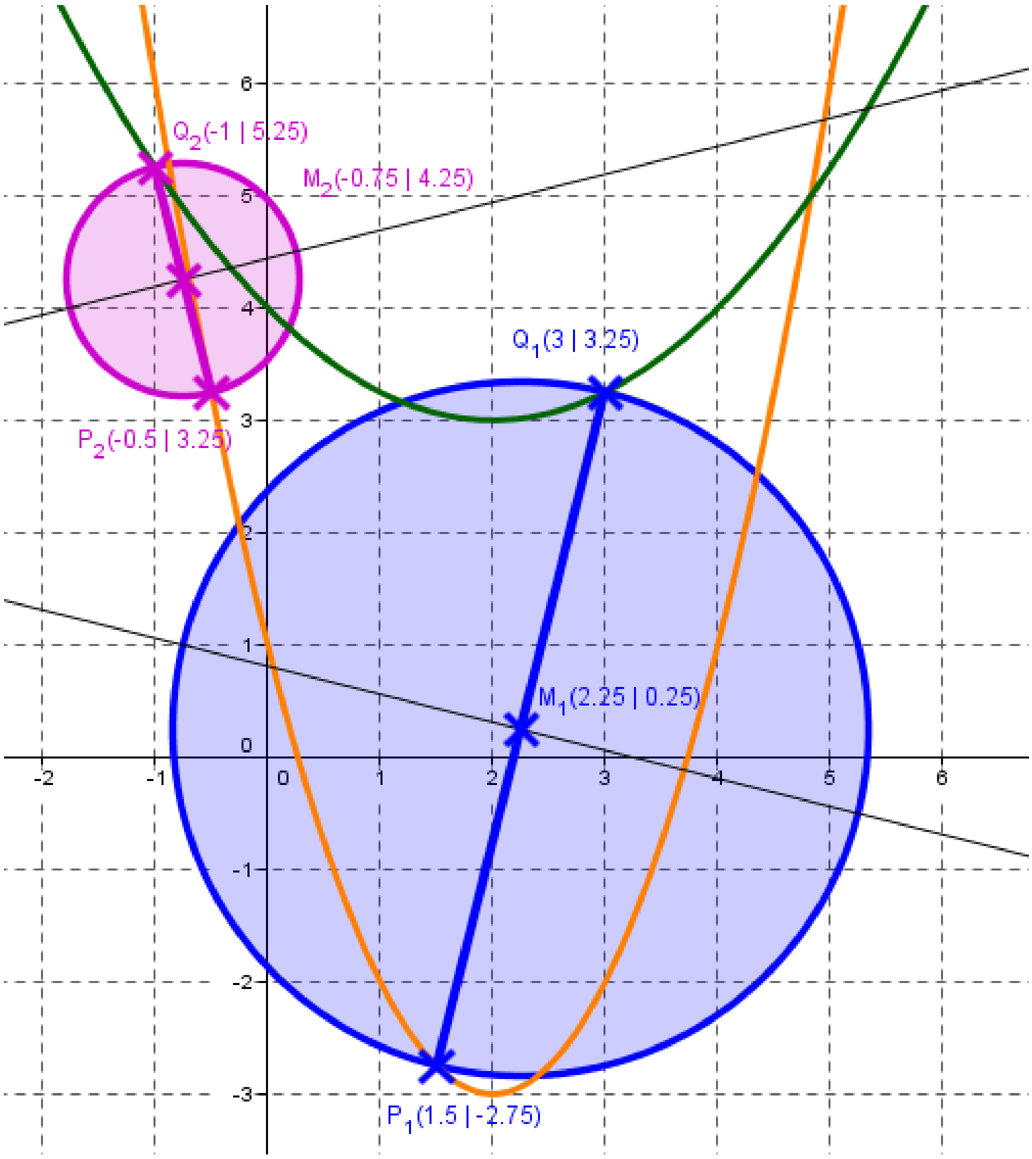
x	-2,0	-1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0
y	7,00	5,25	4,00	3,25	3,00	3,25	4,00	5,25	7,00

B 1.2

$P_1(1{,}5 \mid -2{,}75)$, $Q_1(3 \mid 3{,}25)$ $P_2(-0{,}5 \mid 3{,}25)$, $Q_2(-1 \mid 5{,}25)$

$\overline{P_1Q_1} = \sqrt{[3 - 1{,}5]^2 + [3{,}25 - (-2{,}75)]^2)}$ LE

$\Leftrightarrow \overline{P_1Q_1} = 6{,}18$ LE



B 1.3

x-Wert: 2x laut 1.2

y-Wert: $0{,}25(2x)^2 - 2x + 4 = x^2 - 2x + 4$

$\Rightarrow Q_n(2x \mid x^2 - 2x + 4)$

B 1.4

$\overline{P_nQ_n} = \sqrt{[2x - x]^2 + [x^2 - 2x + 4 - (x^2 - 4x + 1)]^2}$ LE

$\Leftrightarrow \overline{P_nQ_n} = \sqrt{x^2 + [2x + 3]^2}$ LE $= \sqrt{x^2 + 4x^2 + 6x + 6x + 9}$ LE

$\Leftrightarrow \overline{P_nQ_n} = \sqrt{5x^2 + 12x + 9}$ LE

B 1.5

$\overline{P_nQ_n}^2 = 5x^2 + 12x + 9$

$\Leftrightarrow \overline{P_nQ_n}^2 = 5(x^2+2{,}4x+1{,}2^2-1{,}2^2)+9$

$\Leftrightarrow \overline{P_nQ_n}^2 = 5(x+1{,}2)^2 + 1{,}8$

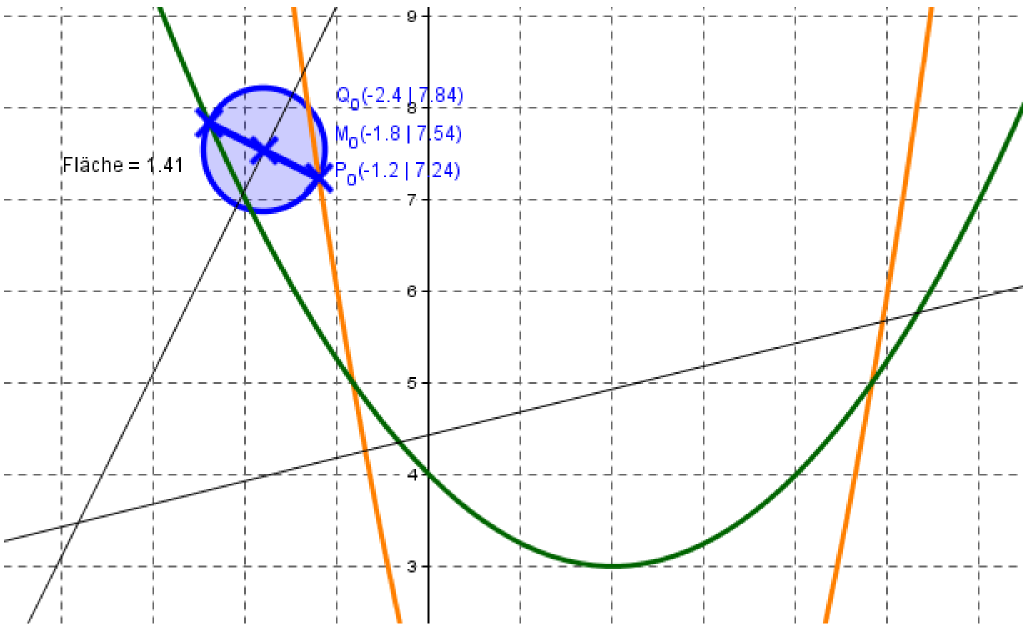
Damit wird für $x = -1{,}2$ der Durchmesser minimal.

Damit ist $A_{min} = (\sqrt{5x^2 + 12x + 9} : 2)^2 \cdot \pi$

$\Leftrightarrow [5\cdot(-1{,}2)^2+12\cdot(-1{,}2)+9]:4 \cdot \pi$

$\Leftrightarrow 0{,}45 \cdot \pi = 1{,}41$ (cm²)

$P_0(-1{,}2 \mid 7{,}24)$, $Q_0(-2{,}4 \mid 3{,}04)$



B 1.6

$u = d \cdot \pi$

$\Leftrightarrow \sqrt{1{,}8} \cdot \pi = \sqrt{5x^2 + 12x + 9} \cdot \pi$

$\Leftrightarrow 1{,}8 = 5x^2 + 12x + 9$

$\Leftrightarrow 5x^2 + 12x + 7{,}2 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{(12)^2 - 4\cdot5\cdot7{,}2}}{10}$$

$$= \frac{-12 \pm \sqrt{0}}{10} \Rightarrow x = -1{,}2 \quad \mathbb{L}=\{-1{,}2\}$$

Aufgabe B2

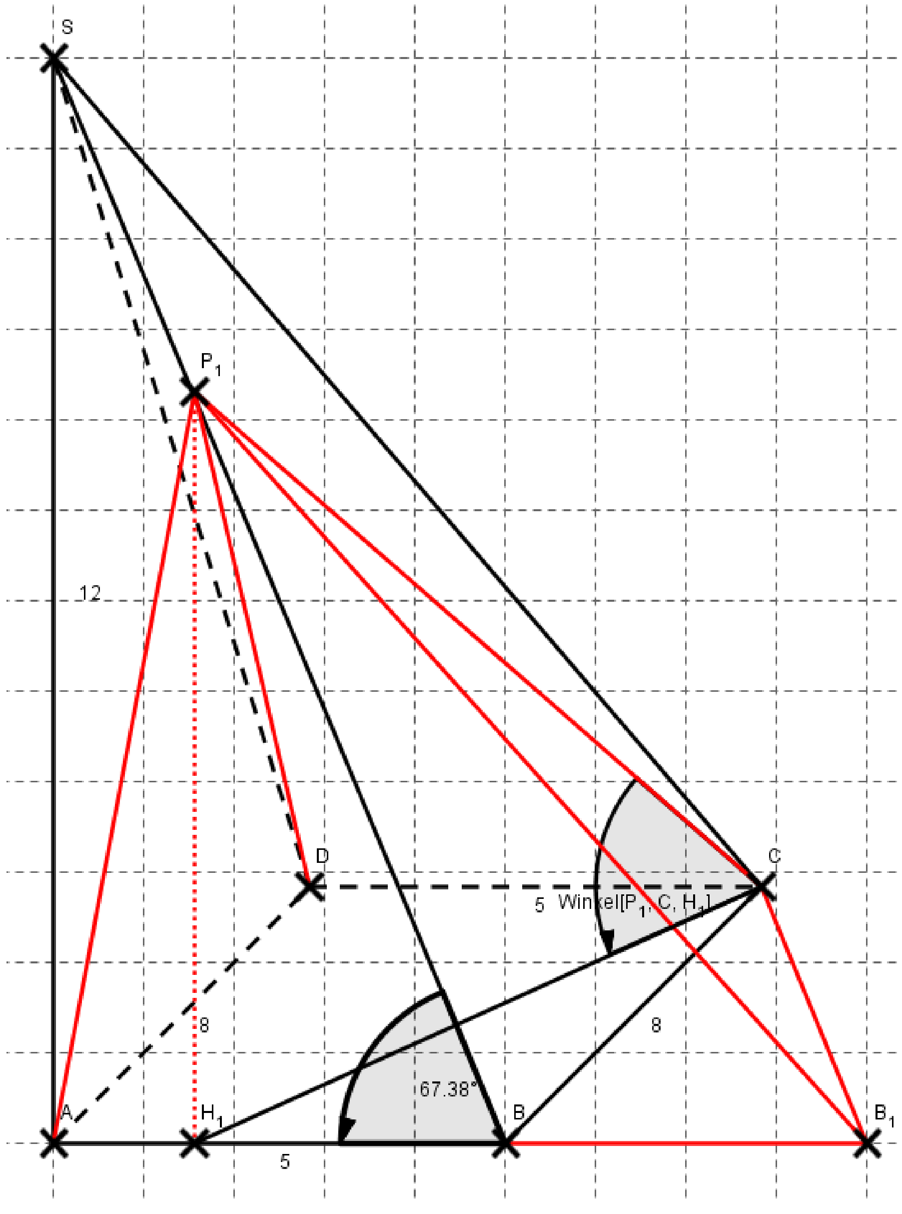
B 2.1

$\overline{BS}^2 = \overline{AB}^2 + \overline{AS}^2$

$\Leftrightarrow \overline{BS}^2 = (5\text{ cm})^2 + (12\text{ cm})^2$

$\Leftrightarrow \overline{BS}^2 = 169\text{ cm}^2$

$\Leftrightarrow \overline{BS} = 13\text{ cm}$



B 2.2 rote Pyramide

B 2.3

$\overline{AS}^2 = \overline{AB}^2 + \overline{BS}^2 - 2 \cdot \overline{AB} \cdot \overline{BS} \cdot \cos\sphericalangle SBA$

$\Leftrightarrow \cos\sphericalangle SBA = \dfrac{\overline{AS}^2 - \overline{AB}^2 - \overline{BS}^2}{-2 \cdot \overline{AB} \cdot \overline{BS}}$

$\Leftrightarrow \cos\sphericalangle SBA = \frac{12^2-5^2-13^2}{-2\cdot 5\cdot 13} = 0,38$

$\Leftrightarrow \sphericalangle SBA = 67,38°$

$\sin\sphericalangle SBA = \frac{h(x)}{\overline{BP_n}} = \frac{h(x)}{13 - x}$

$\Leftrightarrow h(x) = [\sin 67,38° \cdot (13 - x)]\,cm$

$\Leftrightarrow h(x) = [\frac{\overline{AS}}{\overline{BS}} \cdot (13 - x)]\,cm = [\frac{12}{13} \cdot (13 - x)]\,cm$

B 2.4

$\cos\sphericalangle P_1BH_1 = \frac{\overline{BH_1}}{\overline{BP_1}} \Leftrightarrow \overline{BH_1} = \cos\sphericalangle P_1BH_1 \cdot \overline{BP_1}$

$\Leftrightarrow \overline{BH_1} = \cos 67,38° \cdot 9\ cm = 3,46\ cm$

$\overline{H_1C}^2 = \overline{BH_1}^2 + \overline{BC}^2$

$\Leftrightarrow \overline{H_1C}^2 = (3,46\ cm)^2 + (8\ cm)^2$

$\Leftrightarrow \overline{H_1C}^2 = 75,97\ cm^2$

$\Leftrightarrow \overline{H_1C} = 8,72\ cm$

$\tan\sphericalangle P_1CH_1 = \frac{\overline{H_1P_1}}{\overline{CH_1}} = \frac{\frac{12}{13} \cdot (13 - 4)\,cm}{8,72\ cm} = 0,95$

$\Leftrightarrow \sphericalangle P_1CH_1 = 43,61°$

B 2.5

$A_G = 0,5(a + c) \cdot h = 0,5(5 + x + 5) \cdot 8\ cm^2 = (40 + 4x)\ cm^2$

$V(x) = \frac{1}{3} \cdot A_G \cdot h = \frac{1}{3} \cdot (40 + 4x) \cdot \frac{12}{13} \cdot (13 - x)\ cm^3$

$\Leftrightarrow V(x) = \frac{12}{39}(520 - 40x + 52x - 4x^2)\ cm^3$

$\Leftrightarrow V(x) = \frac{4}{13}(520 + 12x - 4x^2) = \frac{16}{13}(-x^2 + 3x + 130)\ cm^3$

[4 ausklammern]

B 2.6

$V_{ABCDS} = \frac{1}{3} \cdot A_G \cdot h = \frac{1}{3} \cdot 5 \cdot 8 \cdot 12\ cm^3 = 160\ cm^3$

$\frac{16}{13}(-x^2 + 3x + 130) = 160$

$\Leftrightarrow -x^2 + 3x + 130 = 130$

$\Leftrightarrow -x^2 + 3x = 0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4\cdot(-1)\cdot 0}}{-1}$

$= \frac{-3 \pm \sqrt{9}}{-1} \Rightarrow x_1=0$ und $x_2=6$ $\mathbb{L}=\{6\}$ da $x > 0$ laut 2.2, und sinnvoll wäre es auch nicht, da $x = 0$ die Originalpyramide ist.

Aufgabe B3

B 3.1

B(-3 | -2) und P(0 | 4) liegen auf g. Also:

I $y = m(x - x_P) + y_P$

I $y = m(x + 3) - 2$

$\Leftrightarrow$ $y = mx + 3m - 2$

II $y = m(x - 0) + 4$

II $y = mx + 4$

I = II $mx + 3m - 2 = mx + 4$

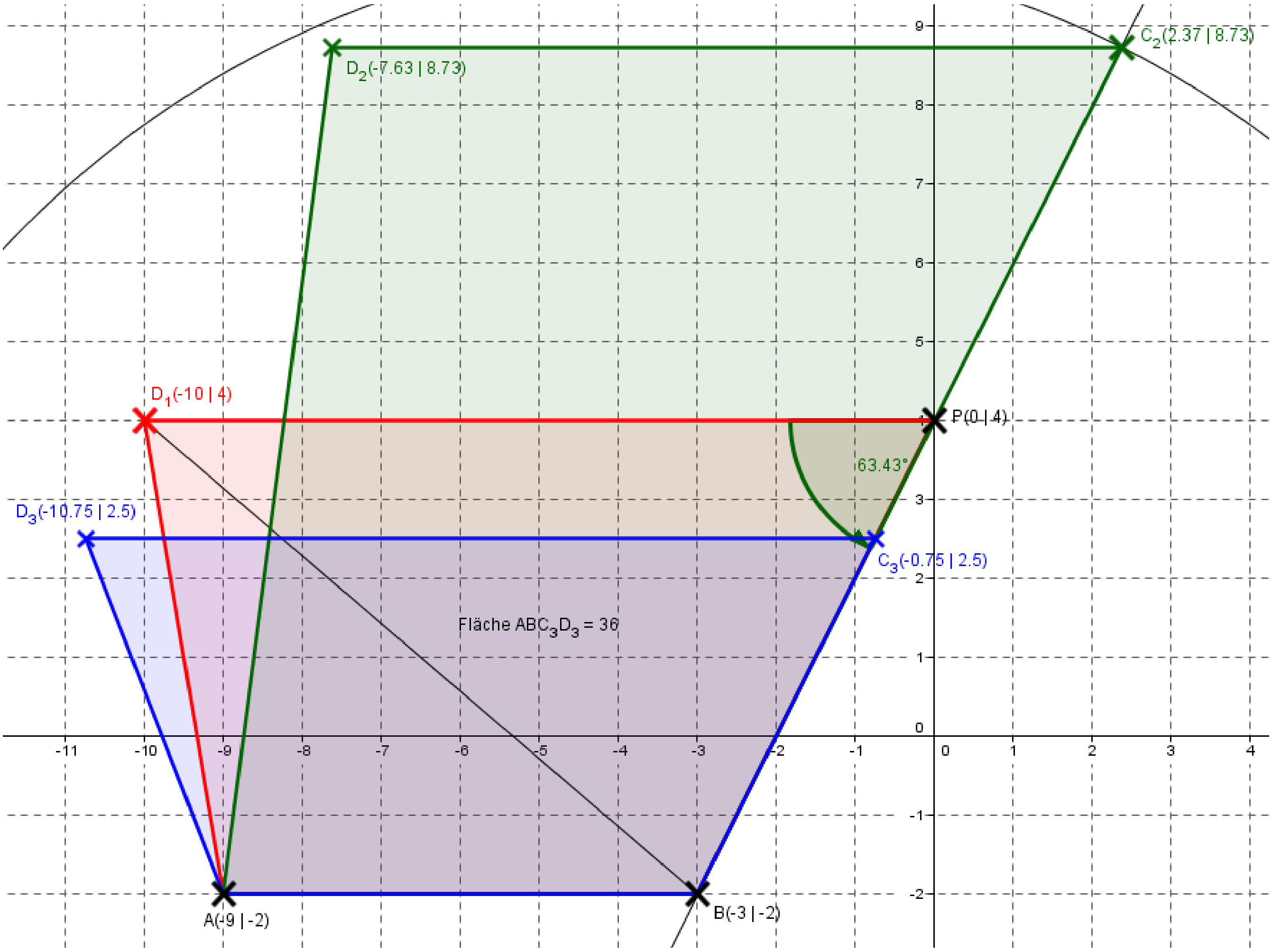
$\Leftrightarrow$ $3m = 6$ $\Leftrightarrow$ $m = 2$

III $y = mx + t = 2x + t$

II = III $2x + 4 = 2x + t$ $\Leftrightarrow$ $t = 4$

Also: g: $y = 2x + 4$

$\tan \sphericalangle D_1C_1B = 2$ $\Leftrightarrow$ $\sphericalangle D_1C_1B = 63{,}43°$



B 3.2

$\overline{BC_2} = \sqrt{[x - (-3)]^2 + [2x + 4 - (-2)]^2}$ LE

$\Leftrightarrow 12^2 = x^2 + 6x + 9 + 4x^2 + 12x + 12x + 36$

$\Leftrightarrow 144 = 5x^2 + 30x + 45$

$\Leftrightarrow 5x^2 + 30x - 99 = 0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-30 \pm \sqrt{30^2 - 4 \cdot 5 \cdot (-99)}}{10}$

$= \frac{-30 \pm \sqrt{2880}}{10} \Rightarrow$ **$x_1 = 2,37$** und $x_2 = -8,37$ $\mathbb{L} = \{2,37\}$ wegen Umlaufsinn.

Also: $C_2(2,37 \mid 8,74)$

B 3.3

$A_{Trapez} = 0,5(a + c) \cdot h$

$\Leftrightarrow 36 = 0,5(6 + 10) \cdot h$

$\Leftrightarrow 36 = 8 \cdot h$

$\Leftrightarrow h = 4,5$ (LE)

Damit ist der y-Wert von C_3 $-2 + 4,5 = 2,5$

$2,5 = 2x + 4$

$\Leftrightarrow -1,5 = 2x$

$\Leftrightarrow x = -0,75$

Damit ist **$C_3(-0,75 \mid 2,5)$**