Abschlussprüfung 1992
an den Realschulen in Bayern
Mathematik II Aufgabengruppe B
Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 28.07.2013

Aufgabe B1 **g: $y = -0{,}5x + 1$** A(-10 | 6) B(2 | 0)

B 1.1 p: $y = 0{,}125x^2 + bx + c$

I $6 = 0{,}125 \cdot (-10)^2 + b \cdot (-10) + c$

$\Leftrightarrow 6 = 12{,}5 - 10b + c$

$\Leftrightarrow c = -6{,}5 + 10b$

II $0 = 0{,}125 \cdot 2^2 + b \cdot 2 + c$

$\Leftrightarrow 0 = 0{,}5 + 2b + c$

$\Leftrightarrow c = -0{,}5 - 2b$

I = II $-6{,}5 + 10b = -0{,}5 - 2b$

$\Leftrightarrow -6 = -12b$

$\Leftrightarrow b = 0{,}5$ in I

I $c = -6{,}5 + 10 \cdot 0{,}5 = -1{,}5$

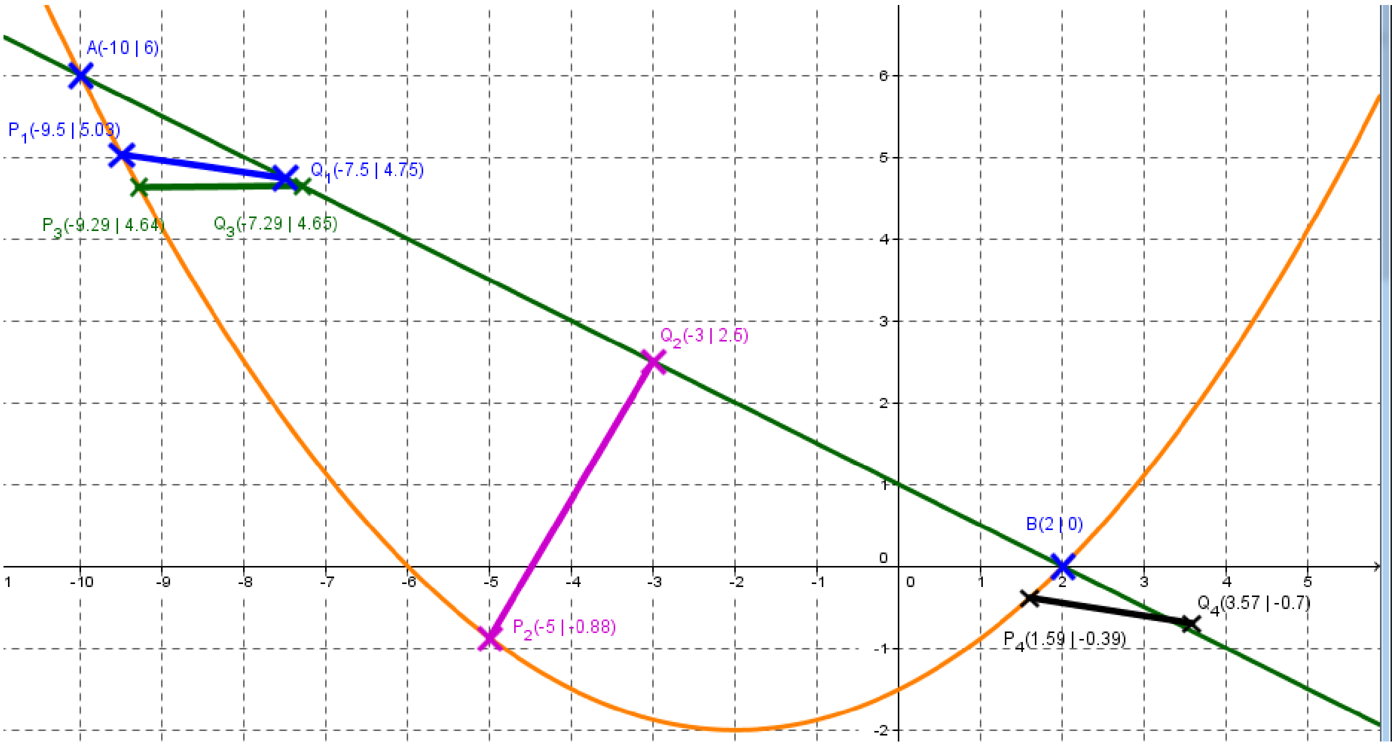
Also: **p: $y = 0{,}125x^2 + 0{,}5x - 1{,}5$** $= \frac{1}{8}x^2 + \frac{1}{2}x - \frac{3}{2}$

$y = 0{,}125(x^2 + 4x + 2^2 - 2^2) - 1{,}5$

$\Leftrightarrow y = 0{,}125(x + 2)^2 - 2$ Also S(-2 | -2)

B 1.2

x	-10,0	-9,0	-8,0	-7,0	-6,0	-5,0	-4,0	-3,0	-2,0	-1,0	0,0	1,0	2,0	3,0
y	6,00	4,13	2,50	1,13	0,00	-0,88	-1,50	-1,88	-2,00	-1,88	-1,50	-0,88	0,00	1,13



B 1.3

$P_1(-9{,}5|5{,}03)$, $Q_1(-7{,}5|3{,}25)$ **$P_2(-5|0{,}875)$, $Q_2(-3|2{,}5)$**

B 1.4

x-Wert: $x + 2$ laut 1.3

y-Wert: $-0,5(x + 2) + 1 = -0,5x = -\frac{1}{2}x$

$\Rightarrow Q_n(x + 2 \mid -\frac{1}{2}x)$

B 1.5

$\overline{P_nQ_n} = \sqrt{[x + 2 - x]^2 + [-0,5x - (0,125x^2 + 0,5x - 1,5)]^2}$ LE

$\Leftrightarrow \overline{P_nQ_n} = \sqrt{[2]^2 + [-x - 0,125x^2 + 1,5]^2}$ LE

$\Leftrightarrow \overline{P_nQ_n} = \sqrt{2^2 + [-\frac{1}{8}x^2 - x + \frac{3}{2}]^2}$ LE

B 1.6

Um $\overline{P_nQ_n}$ minimal werden zu lassen, muss der Wert unter der Wurzel gleich 0 sein.

$0 = -0,125x^2 - x + 1,5$

$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{-1^2 - 4 \cdot (-0,125) \cdot 1,5}}{-0,25}$

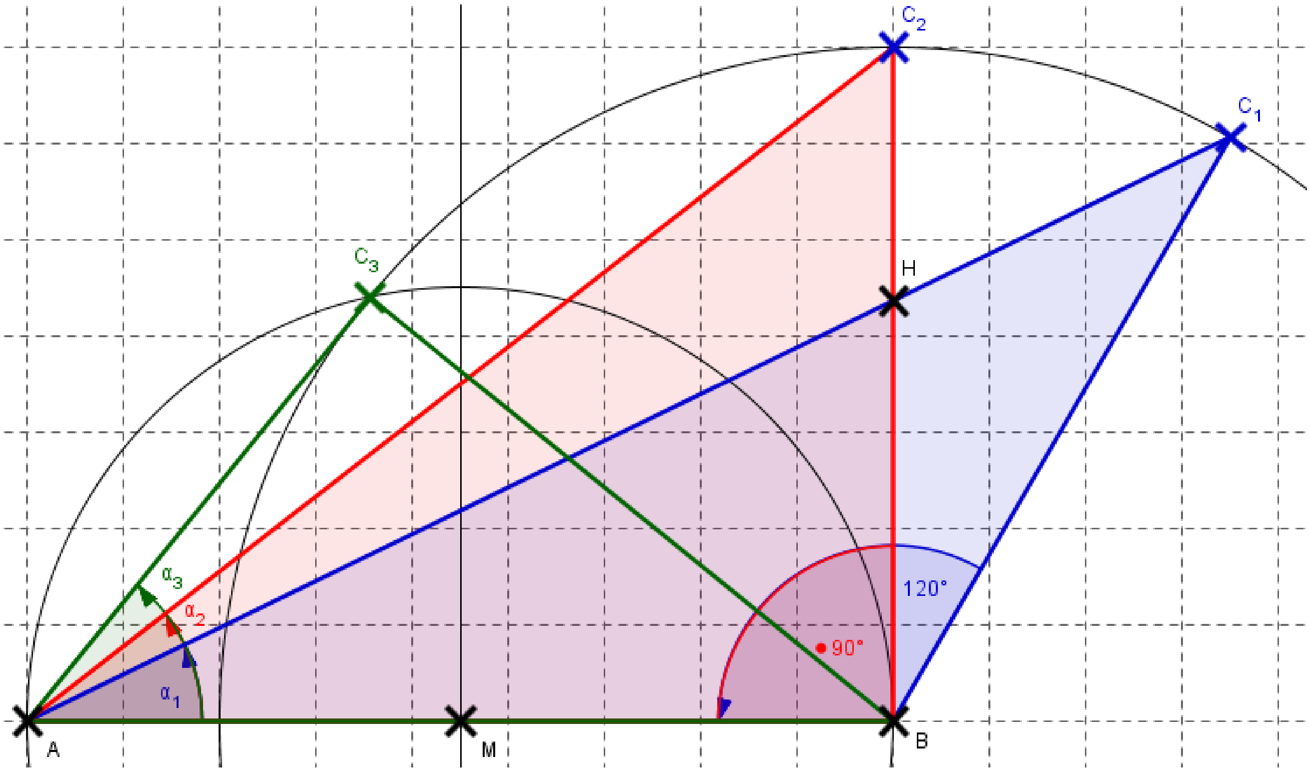
$= \frac{1 \pm \sqrt{1,75}}{-0,25} \Rightarrow x_1 = -9,29$ und $x_2 = 1,29$ $\mathbb{L} = \{-9,29;\ 1,59\}$

[Kontrolle]

$P_3(-9,29|4,64)$, $Q_3(-7,29|4,65)$ $P_4(1,59|0,39)$, $Q_4(3,59|0,21)$

Aufgabe B2

B 2.1



B 2.2

$\overline{AC_1}^2 = \overline{AB}^2 + \overline{BC_1}^2 - 2 \cdot \overline{AB} \cdot \overline{BC_1} \cdot \cos\sphericalangle C_1BA$

$\Leftrightarrow \overline{AC_1}^2 = (9\ \text{cm})^2 + (7\ \text{cm})^2 - 2 \cdot 9\ \text{cm} \cdot 7\ \text{cm} \cdot \cos 120°$

$\Leftrightarrow \overline{AC_1}^2 = 193\ \text{cm}^2$

$\Leftrightarrow \overline{AC_1} = 13{,}89\ \text{cm}$

$\frac{\overline{AC_1}}{\sin \sphericalangle C_1BA} = \frac{\overline{BC_1}}{\sin \sphericalangle BAC_1} \Leftrightarrow \sin \sphericalangle BAC_1 = \frac{\overline{BC_1} \cdot \sin \sphericalangle C_1BA}{\overline{AC_1}}$

$\Leftrightarrow \sin \sphericalangle BAC_1 = \frac{7\ \text{cm} \cdot \sin 120°}{13{,}89\ \text{cm}} = 0{,}44$

$\Leftrightarrow \sphericalangle BAC_1 = 25{,}88°$ (154,12° nicht möglich wegen bereits vorhandenem 120°-Winkel im Dreieck)

$\tan \sphericalangle BAC_2 = \frac{\overline{BC_2}}{\overline{AB}} = \frac{7\ \text{cm}}{9\ \text{cm}} = 0{,}78 \quad \Leftrightarrow \sphericalangle BAC_2 = 37{,}87°$

B 2.3

$[AC_3]$ ist Tangente an den Kreis. Konstruktion über $m_{[AB]}$ und Thaleskreis. Damit entsteht bei **C_3** ein rechter Winkel (Tangente steht senkrecht auf dem Radius).

$\sin \sphericalangle BAC_3 = \frac{\overline{BC_3}}{\overline{AB}} = \frac{7\ \text{cm}}{9\ \text{cm}} = 0{,}78 \Leftrightarrow \sphericalangle BAC_3 = 51{,}06°$ (128,94° nicht möglich wegen bereits vorhandenem 90°-Winkel im Dreieck).

B 2.4

$A_{KreissektorBC1C2} = r^2 \cdot \pi \cdot \frac{30°}{360°} = 49\ cm^2 \cdot \pi \cdot \frac{30°}{360°} = 12{,}83\ cm^2$

Dreieck ABC_1:

$\sphericalangle AC_1B = 180° - 120° - 25{,}88° = 34{,}12°$

Dreieck BC_1H:

$\sphericalangle HC_1B = 180° - 30° - 34{,}12° = 115{,}88°$

$$\frac{\overline{BH}}{\sin \sphericalangle AC_1B} = \frac{\overline{BC_1}}{\sin \sphericalangle BHC_1} \Leftrightarrow \overline{BH} = \frac{\overline{BC_1} \cdot \sin \sphericalangle HC_1B}{\sin \sphericalangle BHC_1}$$

$$\Leftrightarrow \overline{BH} = \frac{7\ cm \cdot \sin 34{,}12°}{\sin 115{,}88°} = 4{,}36\ cm$$

$A_{BC1H} = 0{,}5 \cdot \overline{BH} \cdot \overline{BC_1} \cdot \sin \sphericalangle C_1BH$

$\Leftrightarrow A_{BC1H} = 0{,}5 \cdot 4{,}36\ cm \cdot 7\ cm \cdot \sin 30° = 7{,}63\ cm^2$

$A_{Rest} = 12{,}83\ cm^2 - 7{,}63\ cm^2 = 5{,}2\ cm^2$

Dreieck AHC_2:

$\sphericalangle AC_2B = 180° - 90° - 37{,}87° = 52{,}13°$

$\overline{C_2H} = \overline{BC_2} - \overline{BH} = 7\ cm - 4{,}36\ cm = 2{,}64\ cm$

$\overline{AC_2}^2 = \overline{BC_2}^2 + \overline{AB}^2 = (7\ cm)^2 + (9\ cm)^2 = 130\ cm^2$

$\Leftrightarrow \overline{AC_2} = 11{,}4\ cm$

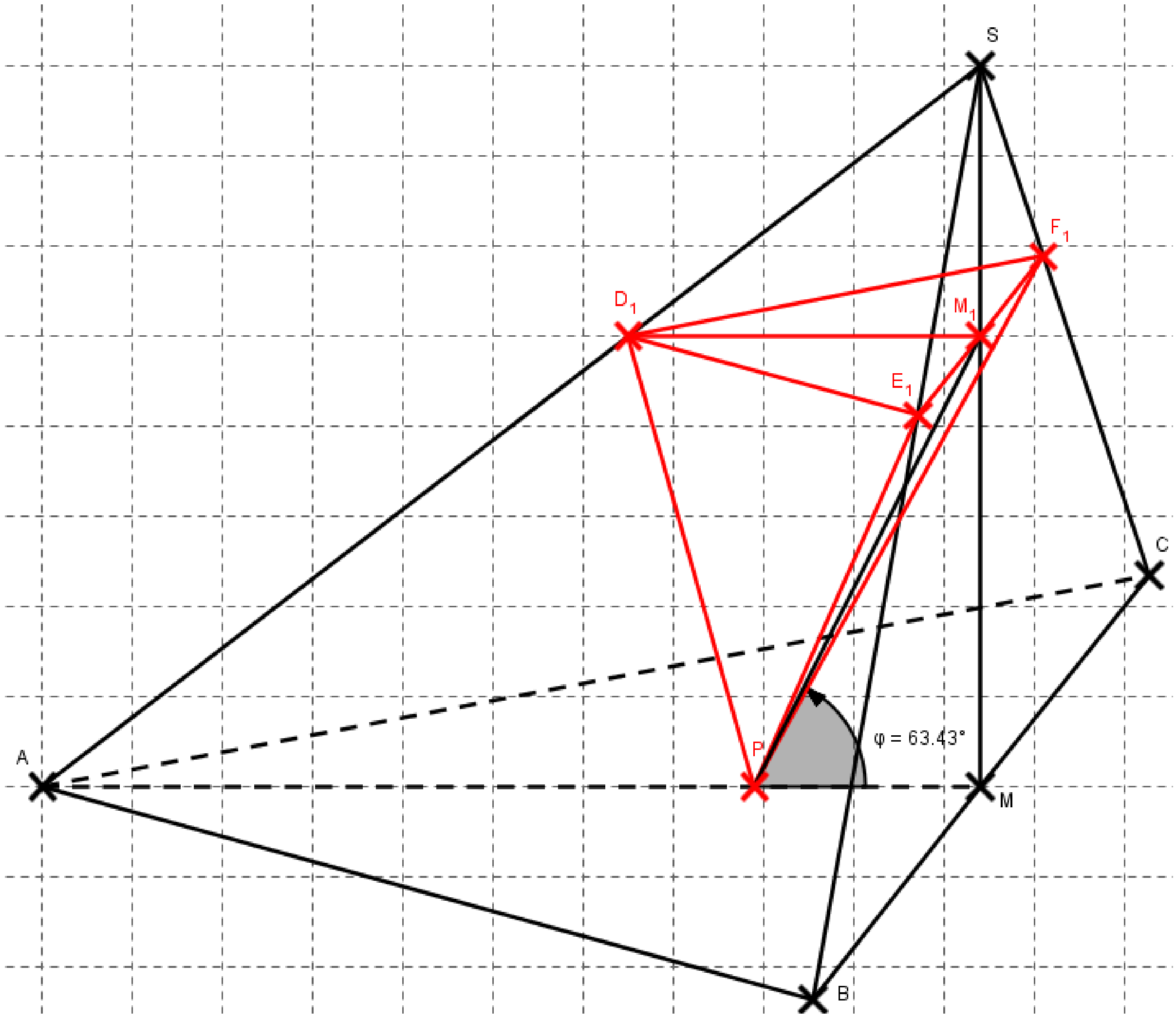
$A_{AHC2} = 0{,}5 \cdot \overline{C_2H} \cdot \overline{AC_2} \cdot \sin \sphericalangle AC_2B$

$\Leftrightarrow A_{BC1H} = 0{,}5 \cdot 2{,}64\ cm \cdot 11{,}4\ cm \cdot \sin 52{,}13° = 11{,}88\ cm^2$

$A_{gesuchteFläche} = 5{,}2\ cm^2 + 11{,}88\ cm^2 = 17{,}08\ cm^2$

Aufgabe B3

B 3.1



$$\overline{AM}^2 = \overline{AB}^2 - \left(\frac{\overline{BC}}{2}\right)^2 = (12\ \text{cm})^2 - (6\ \text{cm})^2 = 108\ \text{cm}^2$$

$$\Leftrightarrow \overline{AM} = 10{,}39\ \text{cm}$$

B 3.2

rote Pyramide

B 3.3

$$\tan \sphericalangle MPM_1 = \frac{\overline{MM_1}}{\overline{MP}} = \frac{5\ \text{cm}}{2{,}5\ \text{cm}} = 2 \quad \Leftrightarrow \sphericalangle MPM_1 = 63{,}43°$$

$$\overline{PM_1}^2 = \overline{MP}^2 + \overline{MM_1}^2 = (2{,}5\ \text{cm})^2 + (5\ \text{cm})^2 = 31{,}25\ \text{cm}^2$$

$$\Leftrightarrow \overline{PM_1} = 5{,}59\ \text{cm}$$

B 3.4

$$\frac{\overline{BC}}{\overline{E_1F_1}} = \frac{\overline{SM}}{\overline{SM_1}} \Leftrightarrow \overline{E_1F_1} = \frac{\overline{BC}\cdot\overline{SM_1}}{\overline{SM}} = \frac{12\text{ cm}\cdot 3\text{ cm}}{8\text{ cm}} = 4{,}5\text{ cm}$$

$$\overline{D_1M_1}^2 = \overline{E_1D_1}^2 - \left(\frac{\overline{E_1F_1}}{2}\right)^2 = (4{,}5\text{ cm})^2-(2{,}25\text{ cm})^2 = 15{,}19\text{ cm}^2$$

$$\Leftrightarrow \overline{D_1M_1} = 3{,}9\text{ cm}$$

$$A_{E1F1D1P} = \frac{1}{3}\cdot A_G\cdot h = \frac{1}{3}\cdot 0{,}5\cdot\overline{D_1M_1}\cdot\overline{E_1F_1}\cdot\overline{MM_1}$$

$$\Leftrightarrow A_{E1F1D1P} = \frac{1}{6}\cdot 3{,}9\text{ cm}\cdot 4{,}5\text{ cm}\cdot 5\text{ cm} = 14{,}63\text{ cm}^3$$

3.5

Dreieck PMM_1:

$\sphericalangle PM_1M = 180° - 63{,}43° - 90° = 26{,}57°$

$\sphericalangle D_1M_1M = 90° - 26{,}57° = 63{,}43°$

$$\overline{D_1P}^2 = \overline{D_1M_1}^2 + \overline{PM_1}^2 - 2\cdot\overline{D_1M_1}\cdot\overline{PM_1}\cdot\cos\sphericalangle D_1M_1M$$

$$\Leftrightarrow \overline{D_1P}^2 = (3{,}9^2 + 5{,}59^2 - 2\cdot 3{,}9\cdot 5{,}59\cdot\cos 63{,}43°)\text{ cm}^2$$

$$\Leftrightarrow \overline{D_1P}^2 = 26{,}96\text{ cm}^2$$

$$\Leftrightarrow \overline{D_1P} = 5{,}19\text{ cm}^2$$

B 3.5

$\sphericalangle PD_1E_1 = \sphericalangle PD_1M_1$

$$\tan\sphericalangle PD_1M_1 = \frac{\overline{M_1P}}{\overline{D_1M_1}} = \frac{5{,}59\text{ cm}}{3{,}9\text{ cm}} = 1{,}43 \Leftrightarrow \sphericalangle PD_1E_1 = 55{,}1°$$

$$\overline{PE_1}^2 = \overline{E_1D_1}^2 + \overline{D_1P}^2 - 2\cdot\overline{E_1D_1}\cdot\overline{D_1P}\cdot\cos\sphericalangle PD_1E_1$$

$$\Leftrightarrow \overline{PE_1}^2 = (4{,}5^2 + 5{,}19^2 - 2\cdot 4{,}5\cdot 5{,}19\cdot\cos 55{,}1°)\text{ cm}^2$$

$$\Leftrightarrow \overline{PE_1}^2 = 20{,}46\text{ cm}^2$$

$$\Leftrightarrow \overline{PE_1} = 4{,}52\text{ cm}^2$$

$$\frac{\overline{D_1P}}{\sin\sphericalangle D_1E_1P} = \frac{\overline{PE_1}}{\sin\sphericalangle PD_1E_1} \Leftrightarrow \sin\sphericalangle D_1E_1P = \frac{\overline{D_1P}\cdot\sin\sphericalangle PD_1E_1}{\overline{PE_1}}$$

$$\Leftrightarrow \sin\sphericalangle D_1E_1P = \frac{5{,}19\text{ cm}\cdot\sin 55{,}1°}{4{,}52\text{ cm}} = 0{,}94$$

$\Leftrightarrow \sphericalangle D_1E_1P = 70{,}34°$ (109,66° nicht möglich, siehe Zeichnung)