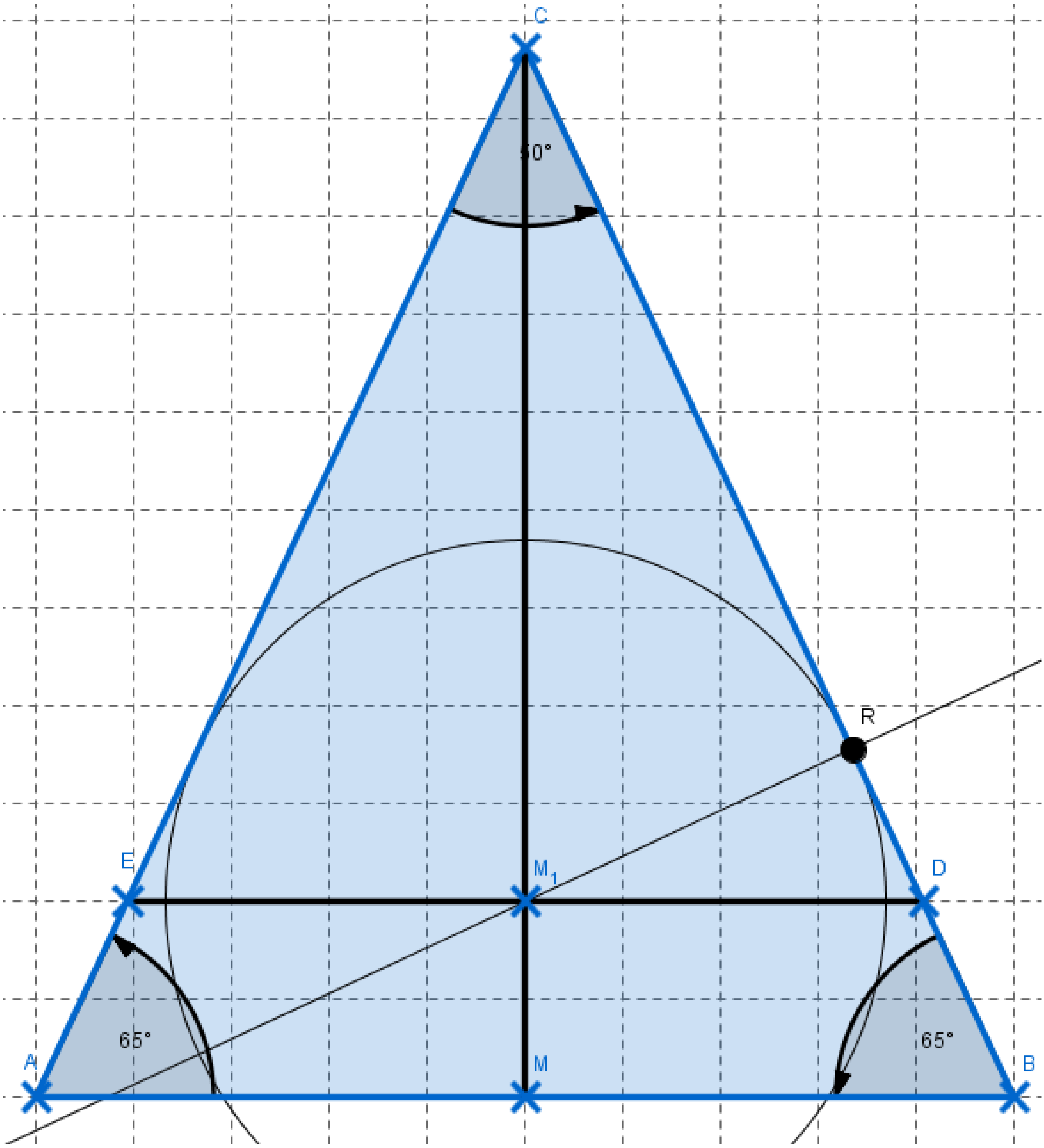

C
50°
R
E
M1
D
65°
65°
A
M
B

Abschlussprüfung 1999
an den Realschulen in Bayern

Mathematik II — Aufgabengruppe B

Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 04.09.2013

Aufgabe B1

B 1.1 **g: $y = 0{,}5x + 2$**

p_1: $y = 0{,}25x^2 + bx + c$ R(0|2) T(10|7)

I $2 = 0 + 0 + c$

$\Leftrightarrow$ $c = 2$

II $7 = 0{,}25 \cdot (10^2) + 10b + c$

$\Leftrightarrow$ $7 = 25 + 10b + 2$

$\Leftrightarrow$ $10b = -20$

$\Leftrightarrow$ $b = -2$

Also: **p_1: $y = 0{,}25x^2 - 2x + 2$**

x	-1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0
y	4,25	2,00	0,25	-1,00	-1,75	-2,00	-1,75	-1,00	0,25	2,00	4,25	7,00

B 1.2

$A_1(1|2{,}5)$ $B_1(1|0{,}25)$ $C_1(3{,}68|3{,}84)$

$A_2(5{,}5|4{,}75)$ $B_2(5{,}5|-1{,}44)$ $C_2(8{,}19|6{,}09)$

$\tan \alpha' = 0{,}5 \Leftrightarrow \alpha' = 26{,}66° \Rightarrow \alpha = 26{,}66° + 90° = 116{,}57°$

B 1.3

$\overline{A_nB_n} = \sqrt{(x - x)^2 + (0{,}5x + 2 - 0{,}25x^2 + 2x - 2)^2}$ cm

$\Leftrightarrow \overline{A_nB_n} = \sqrt{(-0{,}25x^2 + 2{,}5x)^2}$ cm

$\Leftrightarrow \overline{A_nB_n} = (-0{,}25x^2 + 2{,}5x)$ cm

$-0{,}25x^2 + 2{,}5x = 3$

$\Leftrightarrow -0{,}25x^2 + 2{,}5x - 3 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2{,}5 \pm \sqrt{2{,}5^2 - 4 \cdot (-0{,}25) \cdot (-3)}}{-0{,}5}$$

$$= \frac{-2{,}5 \pm \sqrt{3{,}25}}{-0{,}5} \Rightarrow x_1 = 1{,}39 \text{ und } x_2 = 8{,}61 \Rightarrow \mathbb{L} = \{1{,}39;\ 8{,}61\}$$

$A_3(1{,}39|2{,}7)$ $A_4(8{,}61|6{,}31)$

B 1.4

$$\sin \sphericalangle C_nA_nF_n = \frac{\overline{C_nF_n}}{\overline{C_nA_n}} \Leftrightarrow \overline{C_nF_n} = \overline{C_nA_n} \cdot \sin \sphericalangle C_nA_nF_n$$

$\Leftrightarrow \overline{C_nF_n} = 3 \text{ cm} \cdot \sin(180° - 116{,}57°) = 2{,}68$ cm

B 1.5

$\overline{A_nF_n}^2 = \overline{C_nA_n}^2 - \overline{C_nF_n}^2 = (3^2 - 2{,}68^2)\ cm^2 = 1{,}82\ cm^2$

$\Leftrightarrow \overline{A_nF_n} = 1{,}35$

$\overrightarrow{A_nC_n} = \begin{pmatrix} 2{,}68 \\ 1{,}35 \end{pmatrix}$

$\overrightarrow{A_nB_n} = \begin{pmatrix} 0 \\ 0{,}25x^2 - 2x + 2 - 0{,}5x - 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}25x^2 - 2{,}5x \end{pmatrix}$

$A(x) = 0{,}5 \cdot \begin{vmatrix} 0 & 2{,}68 \\ 0{,}25x^2 - 2{,}5x & 1{,}35 \end{vmatrix}$ FE

$\Leftrightarrow A(x) = [0{,}5 \cdot (-0{,}67x^2 + 6{,}7x)]$ FE

$\Leftrightarrow A(x) = [-0{,}335x^2 + 3{,}35x]$ FE

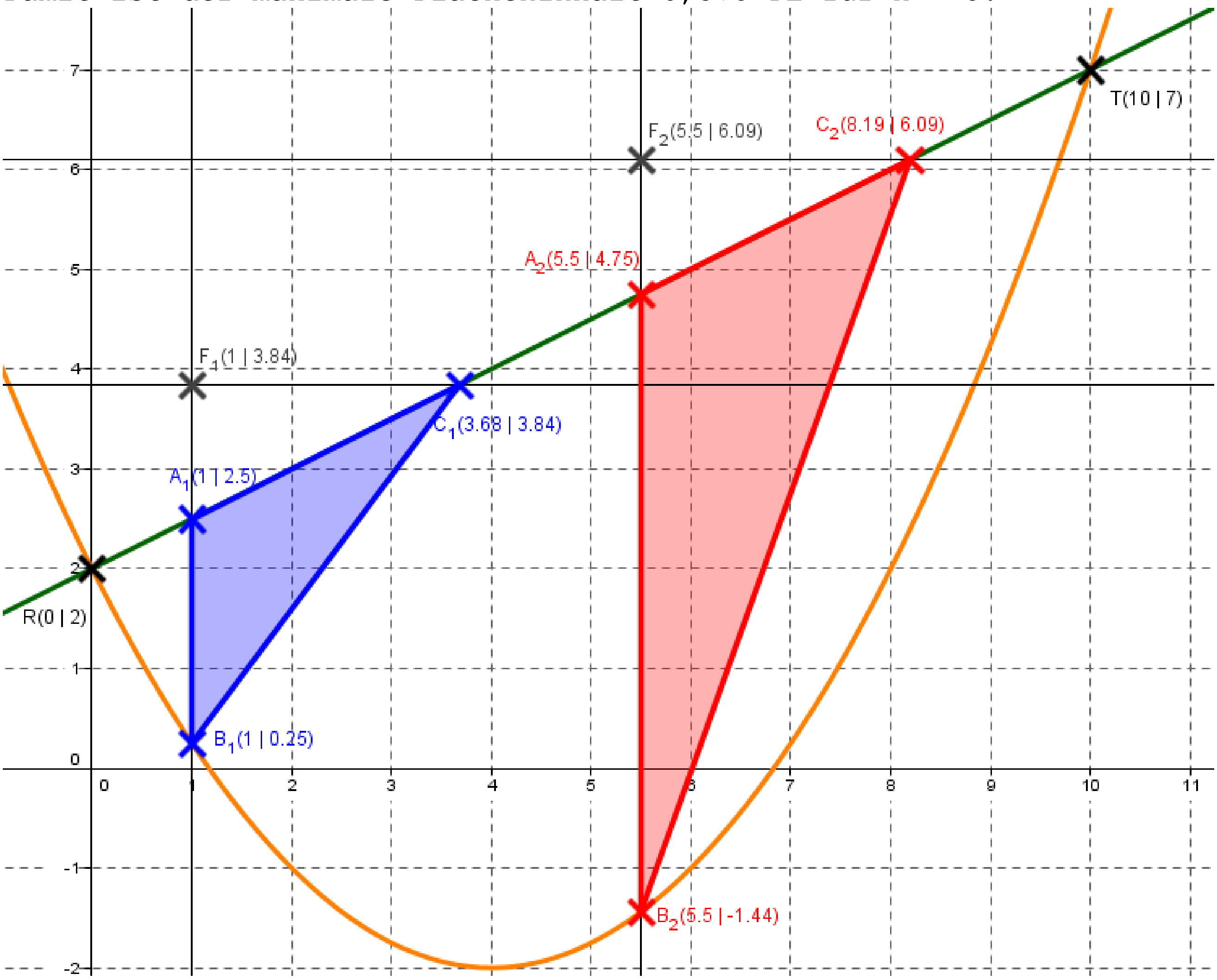
$A(x) = -0{,}335x^2 + 3{,}35x$

$\Leftrightarrow A(x) = -0{,}335(x^2 - 10x)$

$\Leftrightarrow A(x) = -0{,}335(x^2 - 10x + 5^2 - 5^2)$

$\Leftrightarrow A(x) = -0{,}335(x - 5)^2 + 8{,}375$

Damit ist der maximale Flächeninhalt 8,375 FE für x = 5.



Aufgabe B2

B 2.1

$\overline{EA}^2 = \overline{AF}^2 + \overline{FE}^2 = (7{,}5^2 + 8^2)\ cm^2 = 120{,}25\ cm^2$

$\Leftrightarrow\ r = \overline{EA} = 10{,}97\ cm$ also 11 m

$\tan \sphericalangle AEF = \frac{\overline{AF}}{\overline{FE}} = \frac{7{,}5\ cm}{8\ cm} = 0{,}94 \Leftrightarrow \sphericalangle AEF = 43{,}2°$

B 2.2

$b = 2 \cdot r \cdot \pi \cdot \frac{43{,}2° \cdot 2}{360°} = 2 \cdot 11\ m \cdot \pi \cdot \frac{86{,}4°}{360°} = 16{,}6\ m$

$\sphericalangle DEA = 90° - 43{,}2° = 46{,}8°$

$\sphericalangle EAD = 180° - 46{,}8° - 100° = 33{,}2°$

$\frac{\overline{AD}}{\sin \sphericalangle DEA} = \frac{\overline{AE}}{\sin \sphericalangle EDA}$

$\Leftrightarrow\ \overline{AD} = \frac{\overline{AE} \cdot \sin \sphericalangle DEA}{\sin \sphericalangle EDA} = \frac{11\ m \cdot \sin 46{,}8°}{\sin 100°} = 8{,}1\ m$

$\frac{\overline{DE}}{\sin \sphericalangle EAD} = \frac{\overline{AE}}{\sin \sphericalangle EDA}$

$\Leftrightarrow\ \overline{DE} = \frac{\overline{AE} \cdot \sin \sphericalangle EAD}{\sin \sphericalangle EDA} = \frac{11\ m \cdot \sin 33{,}2°}{\sin 100°} = 6{,}1\ m$

$u_{gesamt} = 6{,}1\ m + 6{,}1\ m + 8{,}1\ m + 8{,}1\ m + 16{,}6\ m = 45\ m$

B 2.3

$\sphericalangle EMR = 180° - 46{,}8° - 46{,}8° = 86{,}4°$

$\overline{BD}^2 = \overline{BC}^2 + \overline{CD}^2 - 2 \cdot \overline{BC} \cdot \overline{CD} \cdot \cos \sphericalangle DCB$

$\Leftrightarrow\ \overline{BD}^2 = (8{,}2^2 + 12{,}2^2 - 2 \cdot 8{,}2 \cdot 12{,}2 \cdot \cos 100°)\ m^2 = 250{,}8\ m^2$

$\Leftrightarrow\ \overline{BD} = 15{,}8\ m$

$\frac{\overline{CB}}{\sin \sphericalangle BDE} = \frac{\overline{BD}}{\sin \sphericalangle DCB}$

$\Leftrightarrow\ \sin \sphericalangle BDE = \frac{\overline{CB} \cdot \sin \sphericalangle DCB}{\overline{BD}} = \frac{8{,}1\ m \cdot \sin 100°}{15{,}8\ m} = 0{,}5$

$\Leftrightarrow\ \sphericalangle BDE = 30{,}3°$ ($149{,}7°$ nicht möglich wegen $\sphericalangle ADE = 100°$)

$\frac{\overline{ME}}{\sin \sphericalangle BDE} = \frac{\overline{DE}}{\sin \sphericalangle EMD} \Leftrightarrow \overline{ME} = \frac{\overline{DE} \cdot \sin \sphericalangle BDE}{\sin \sphericalangle EMD}$

$\Leftrightarrow\ \overline{ME} = \frac{6{,}1\ m \cdot \sin 30{,}3°}{\sin(180° - 30{,}3° - 46{,}8°)} = 3{,}2\ m$

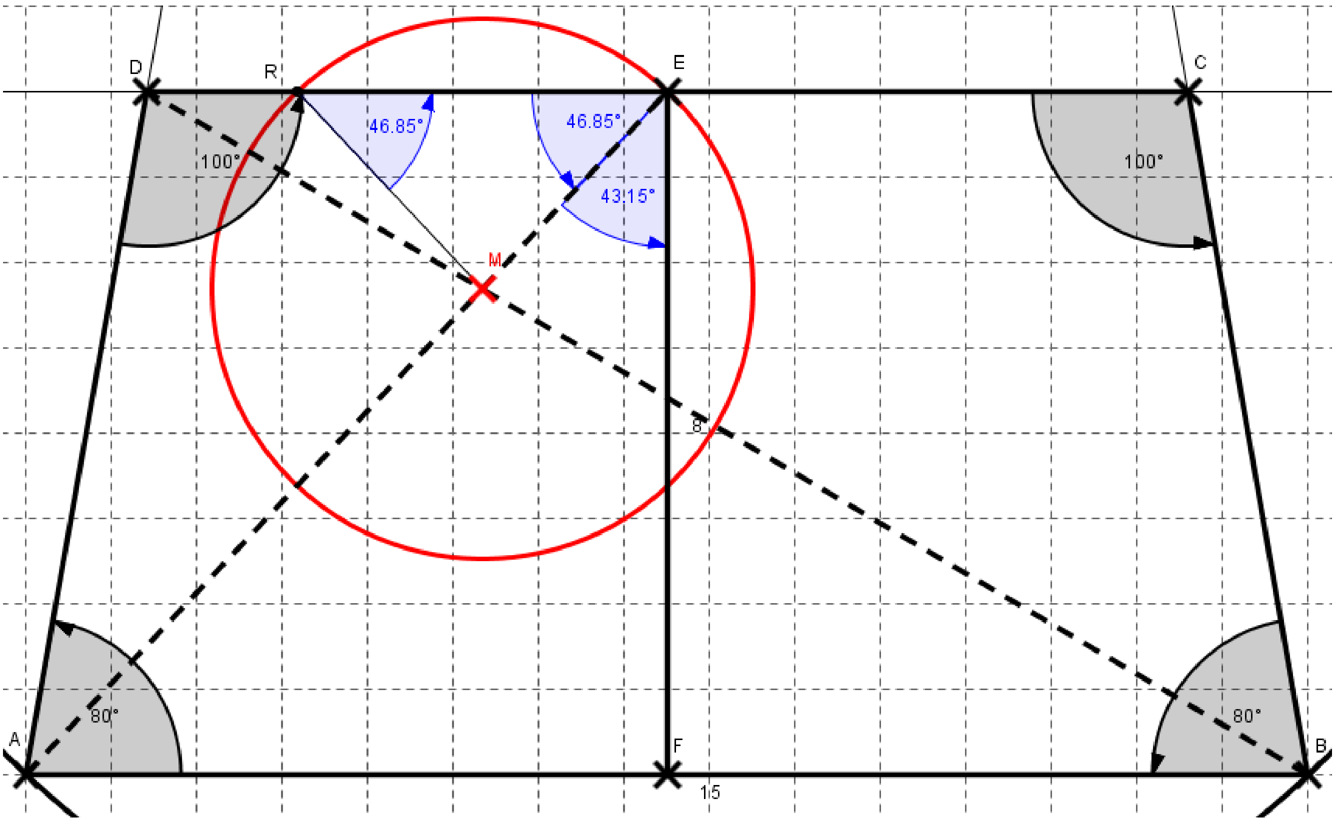
B 2.4

$A_{Sektor} = r^2 \cdot \pi \cdot \frac{\sphericalangle EMR}{360°}$

$\Leftrightarrow A_{Sektor} = (3{,}2\ m)^2 \cdot \pi \cdot \frac{86{,}4°}{360°} = 7{,}7\ m^2$

$A_{MER} = 0{,}5 \cdot r \cdot r \cdot \sin 86{,}4° = 0{,}5 \cdot (3{,}2\ m)^2 \cdot \sin 86{,}4° = 5{,}1\ m^2$

$A_{außen} = 7{,}7\ m^2 - 5{,}1\ m^2 = 2{,}6\ m^2$



Aufgabe B3

B 3.1

$\tan\gamma = \dfrac{\overline{AS}}{\overline{AC}} = \dfrac{9\ \text{cm}}{10\ \text{cm}} = 0{,}9 \quad \Leftrightarrow \gamma = 41{,}99°$

$\overline{CS}^2 = \overline{AS}^2 + \overline{AC}^2 = (9^2 + 10^2)\ \text{cm}^2 = 181\ \text{cm}^2$

$\Leftrightarrow \overline{CS} = 13{,}45\ \text{cm}$

B 3.2

$\sphericalangle MP_1C = 180° - 70° - 41{,}99° = 68{,}01°$

$\dfrac{\overline{CP_1}}{\sin \sphericalangle CMP_1} = \dfrac{\overline{MC}}{\sin \sphericalangle MP_1C}$

$\Leftrightarrow \overline{CP_1} = \dfrac{\overline{MC} \cdot \sin \sphericalangle CMP_1}{\sin \sphericalangle MP_1C} = \dfrac{7\ \text{cm} \cdot \sin 70°}{\sin 68{,}01°} = 7{,}09\ \text{cm}$

B 3.3

$\sin\gamma = \dfrac{h}{\overline{CP_1}} \Leftrightarrow h = \sin\gamma \cdot \overline{CP_1} = \sin 41{,}99° \cdot 7{,}09\ \text{cm} = 4{,}74\ \text{cm}$

$V = \dfrac{1}{3} \cdot A_G \cdot h = \dfrac{1}{3} \cdot 0{,}5 \cdot 7\ \text{cm} \cdot 4\ \text{cm} \cdot 4{,}74\ \text{cm} = 22{,}12\ \text{cm}^2$

B 3.4

$A_{ABCD} = 0{,}5 \cdot e \cdot f = 0{,}5 \cdot 10\ \text{cm} \cdot 8\ \text{cm} = 40\ \text{cm}^2$

$A_{BDP2} = 40\ \text{cm}^2 \cdot 0{,}8 = 32\ \text{cm}^2$

$32\ \text{cm}^2 = 0{,}5 \cdot \overline{BD} \cdot \overline{MP_2}$

$\Leftrightarrow \overline{MP_2} = \dfrac{32\ \text{cm}^2}{0{,}5 \cdot \overline{BD}} = \dfrac{32\ \text{cm}^2}{0{,}5 \cdot 8\ \text{cm}} = 8\ \text{cm}$

$\dfrac{\overline{MP_2}}{\sin\gamma} = \dfrac{\overline{MC}}{\sin \sphericalangle MP_2C}$

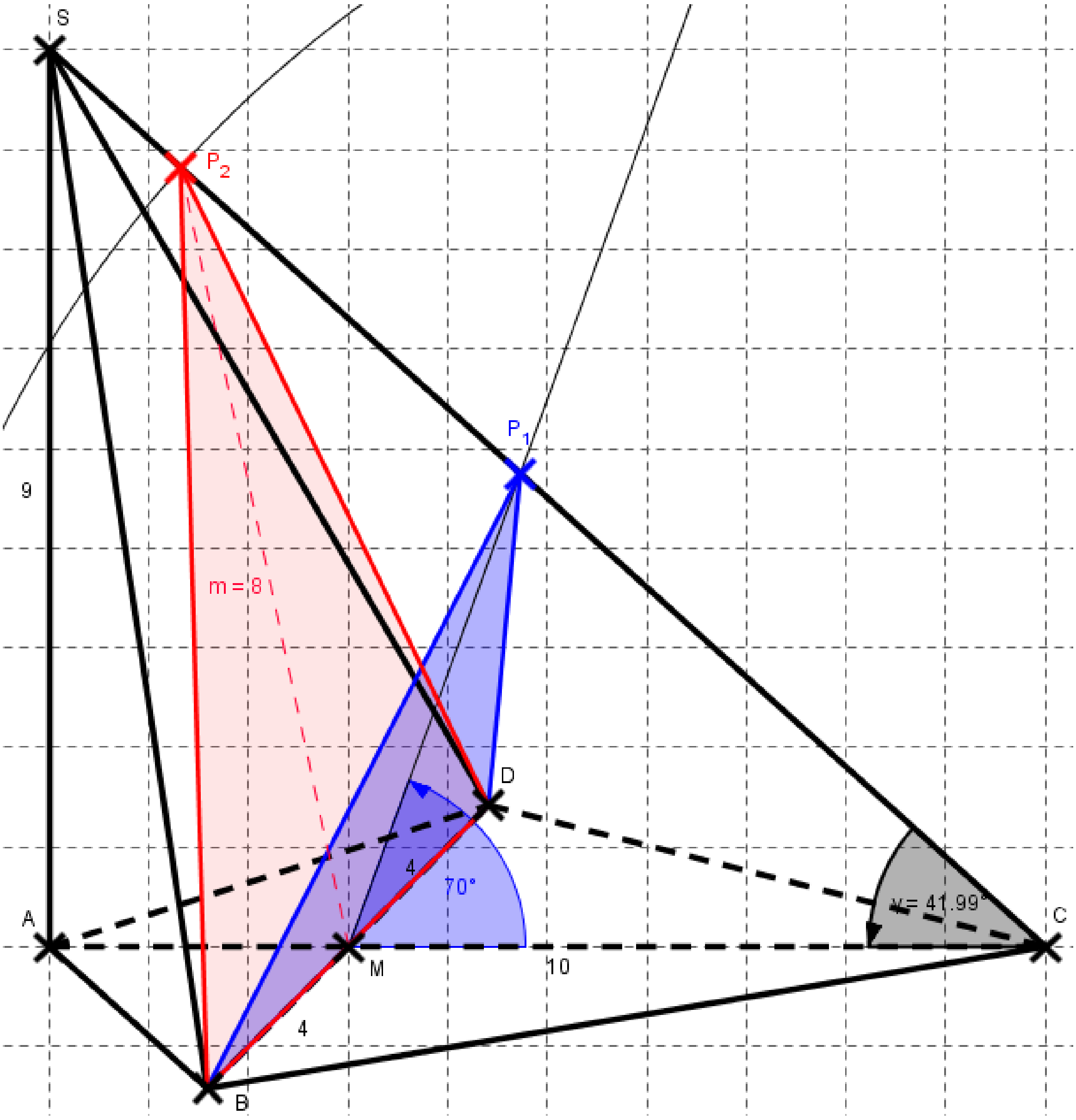
$\Leftrightarrow \sin \sphericalangle MP_2C = \dfrac{\overline{MC} \cdot \sin\gamma}{\overline{MP_2}} = \dfrac{7\ \text{cm} \cdot \sin 41{,}99°}{8\ \text{cm}} = 0{,}59$

$\Leftrightarrow \sphericalangle MP_2C = 35{,}83°$ (144,17° wegen $\gamma = 41{,}99°$ (Innenwinkelsumme) nicht möglich)

$\dfrac{\overline{MP_2}}{\sin\gamma} = \dfrac{x}{\sin \sphericalangle CMP_2} \Leftrightarrow x = \dfrac{\overline{MP_2} \cdot \sin \sphericalangle CMP_2}{\sin\gamma}$

$\Leftrightarrow x = \dfrac{8\ \text{cm} \cdot \sin(180° - 41{,}99° - 35{,}83°)}{\sin 41{,}99°} = 11{,}69\ \text{cm}$

[Zeichnung nur zur Kontrolle]


S
P2
m = 8
P1
9
D
4
70°
A
M
10
C
4
B
γ = 41.99°