Abschlussprüfung 1999
an den Realschulen in Bayern
Mathematik II Aufgabengruppe A
Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 04.09.2013

Aufgabe A1

A 1.1

Bei Parallelverschiebung bleibt a gleich. Scheitelform von p:

$p: y = -0{,}25(x-4)^2 + 2{,}5$

$\Leftrightarrow p: y = -0{,}25(x^2 - 8x + 16) + 2{,}5$

$\Leftrightarrow \mathbf{p: y = -0{,}25x^2 + 2x - 1{,}5}$

$\mathbf{g: y = -x + 7{,}5}$

g Tangente an p? -> Gleichsetzen und D = 0:

$\mathbf{-x + 7{,}5} = \mathbf{-0{,}25x^2 + 2x - 1{,}5}$

$\Leftrightarrow -0{,}25x^2 + 3x - 9 = 0$

$b^2 - 4ac = 9 - 4 \cdot 0{,}25 \cdot 9 = 9 - 9 = 0$

A 1.2

x	-1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0
y	-3,8	-1,5	0,3	1,5	2,3	2,5	2,3	1,5	0,3	-1,5	-3,8

A 1.3

$\mathbf{A_1(-1|-3{,}75)}$ $\mathbf{A_2(0{,}5|-0{,}5625)}$ $\mathbf{B_1(6|-3{,}75)}$ $\mathbf{B_2(6|-0{,}5625)}$

$x < 2$

A 1.4

$-0{,}25x^2 + 2x - 1{,}5 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 1{,}5}}{2 \cdot (-0{,}25)} = \frac{2 \pm \sqrt{2{,}5}}{0{,}5}$$

$x_1 = 7{,}16$ und $x_2 = 0{,}84$ Da $x < 2$: $\mathbb{L} = \{0{,}84\}$

$A_3(0{,}84|0)$ $B_3(6|0)$

$$\overrightarrow{A_3B_3} = \begin{pmatrix} 6-0{,}84 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 5{,}16 \\ 0 \end{pmatrix} \qquad \overrightarrow{A_3C} = \begin{pmatrix} 6-0{,}84 \\ 1{,}5 \end{pmatrix} = \begin{pmatrix} 5{,}16 \\ 1{,}5 \end{pmatrix}$$

$$A_{\Delta A3B3C} = 0{,}5 \begin{vmatrix} 5{,}16 & 5{,}16 \\ 0 & 1{,}5 \end{vmatrix} \text{ FE} = 3{,}87 \text{ FE}$$

A 1.5

$\tan 36° = 0{,}73$

Gerade A_4C: $g: y = m(x-x_p) + y_p$

$\Leftrightarrow y = 0{,}73(x-6) + 1{,}5$

$\Leftrightarrow$ y = 0,73x – 2,88 (gestrichelt in Zeichnung)

Für Schnittpunkt gleichsetzen:

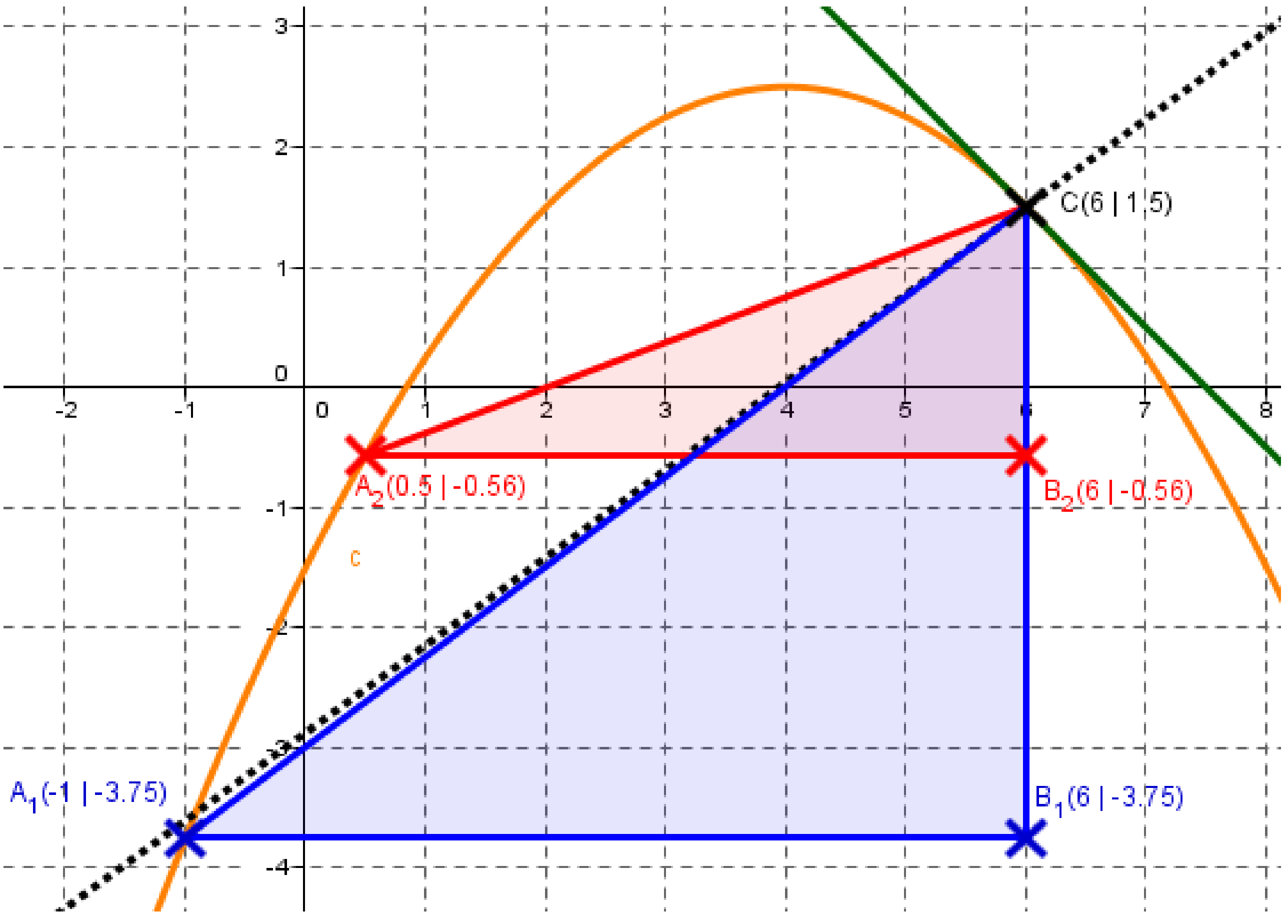
$$0{,}73x - 2{,}88 = \mathbf{-0{,}25x^2 + 2x - 1{,}5}$$

$$\Leftrightarrow -0{,}25x^2 + 1{,}27x + 1{,}38 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1{,}27 \pm \sqrt{1{,}27^2 + 1{,}38}}{2 \cdot (-0{,}25)} =$$

$$\frac{1{,}27 \pm \sqrt{3{,}0658}}{0{,}5}$$

$x_1 = -0{,}96$ und $x_2 = 6{,}04$ Da $x<2$ ist x_1 die Lösung: $\mathbb{L} = \{-0{,}96\}$



Aufgabe A2

A 2.1 und A 2.2

$\overline{PR}^2 = \overline{RS}^2 + \overline{SP}^2 - 2 \cdot \overline{RS} \cdot \overline{SP} \cdot \cos \sphericalangle PSR$

$\Leftrightarrow \overline{PR}^2 = (9{,}5^2 + 13{,}5^2 - 2 \cdot 9{,}5 \cdot 13{,}5 \cdot \cos 68°)\ cm^2 = 176{,}41\ cm^2$

$\Leftrightarrow \overline{PR} = 13{,}28\ cm$ (also 132,8 m)

$\overline{SQ}^2 = \overline{QP}^2 + \overline{SP}^2 = (5{,}2^2 + 13{,}5^2)\ cm^2 = 209{,}29\ cm^2$

$\Leftrightarrow \overline{SQ} = 14{,}47\ cm$

$\tan \sphericalangle PSQ = \frac{\overline{QP}}{\overline{SP}} = \frac{5{,}2\ cm}{13{,}5\ cm} = 0{,}39 \quad \Leftrightarrow \sphericalangle PSQ = 21{,}07°$

$\sphericalangle QSR = 68° - 21{,}07° = 46{,}93°$

$\overline{RQ}^2 = \overline{RS}^2 + \overline{SQ}^2 - 2 \cdot \overline{RS} \cdot \overline{SQ} \cdot \cos \sphericalangle QSR$

$\Leftrightarrow \overline{RQ}^2 = (9{,}5^2 + 14{,}47^2 - 2 \cdot 9{,}5 \cdot 14{,}47 \cdot \cos 46{,}93°)cm^2 = 111{,}88\ cm^2$

$\Leftrightarrow \overline{RQ} = 10{,}58\ cm$ (also 105,8 m)

$\overline{PR}^2 = \overline{RQ}^2 + \overline{QP}^2 - 2 \cdot \overline{RQ} \cdot \overline{QP} \cdot \cos \sphericalangle RQP$

$$\Leftrightarrow \cos \sphericalangle RQP = \frac{\overline{PR}^2 - \overline{RQ}^2 - \overline{QP}^2}{-2 \cdot \overline{RQ} \cdot \overline{PQ}}$$

$$\Leftrightarrow \cos \sphericalangle RQP = \frac{13{,}28^2 - 10{,}58^2 - 5{,}2^2}{-2 \cdot 10{,}58 \cdot 5{,}2} = -0{,}34$$

$\Leftrightarrow \sphericalangle RQP = 109{,}86°$

A 2.3

$\sphericalangle EMF = 360° - 90° - 90° - 68° = 112°$

$b = 2 \cdot r \cdot \pi \cdot \frac{\sphericalangle EMF}{360°} = 2 \cdot 2\ cm \cdot \pi \cdot \frac{112°}{360°} = 3{,}91\ cm$ (also 39,1 m)

A 2.4

$\sphericalangle EMS = 112° : 2 = 56°$

$\cos \sphericalangle EMS = \frac{\overline{ME}}{\overline{MS}} \Leftrightarrow \overline{MS} = \frac{\overline{ME}}{\cos \sphericalangle EMS} = \frac{20\ m}{\cos 56°} = 35{,}8\ m$

$A_{Drache} = 2 \cdot 0{,}5 \cdot \overline{ME} \cdot \overline{MS} \cdot \sin 56°$

$\Leftrightarrow A_{Drache} = 20\ m \cdot 35{,}8\ m \cdot \sin 56° = 593{,}6\ m^2$

$A_{Sektor} = r^2 \cdot \pi \cdot \frac{\sphericalangle EMF}{360°} = (20\ m)^2 \cdot \pi \cdot \frac{112°}{360°} = 390\ m^2$

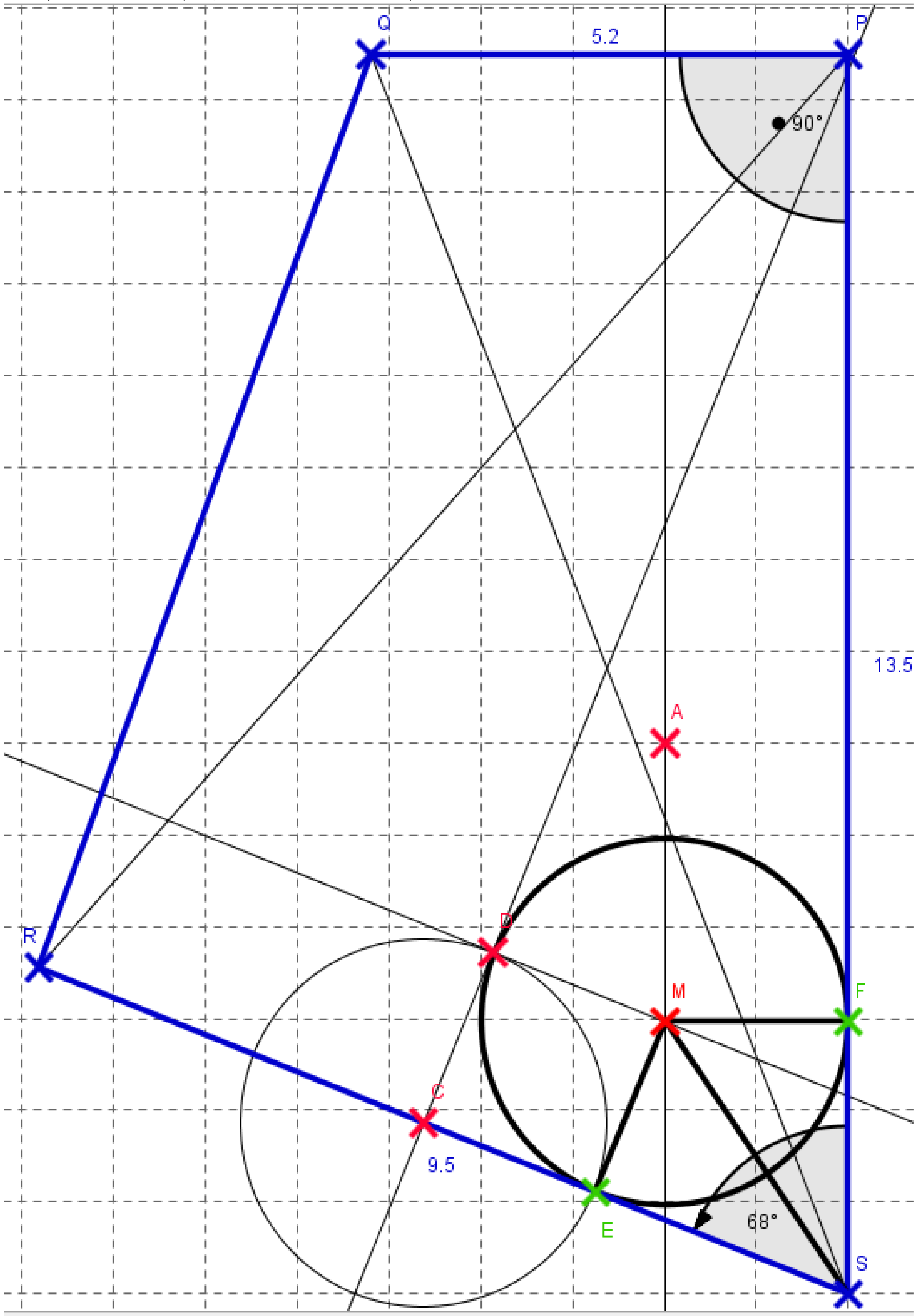
$A_{weggeschnitten} = 593{,}6\ m^2 - 390\ m^2 = 202{,}6\ m^2$

A 2.5 $A_{alles} = 0{,}5 \cdot \overline{RQ} \cdot \overline{QP} \cdot \sin \sphericalangle RQP + 0{,}5 \cdot \overline{RS} \cdot \overline{SP} \cdot \sin \sphericalangle PSR$

$\Leftrightarrow A_{alles} = (0{,}5 \cdot 105{,}8 \cdot 52 \cdot \sin 109{,}86 + 0{,}5 \cdot 95 \cdot 135 \cdot \sin 68°)m^2$

$\Leftrightarrow A_{alles} = 8532{,}8\ m^2$

(202,6 : 8532,8) · 100 % = 2,37 %



Aufgabe A3

A 3.1

$$\tan\beta = \frac{\overline{MC}}{\overline{MB}} \Leftrightarrow \overline{MC} = \tan\beta \cdot \overline{MB} = \tan 65^\circ \cdot 5\text{ cm} = 10{,}72\text{ cm}$$

A 3.2

$$\frac{\overline{MC}}{\overline{CM_1}} = \frac{\overline{AB}}{\overline{ED}} \Leftrightarrow \overline{ED} = \frac{\overline{AB}\cdot\overline{CM_1}}{\overline{MC}} = \frac{10\text{ cm}\cdot 8{,}72\text{ cm}}{10{,}72\text{ cm}} = 8{,}13\text{ cm}$$

$$\sin\sphericalangle RDM_1 = \frac{\overline{M_1R}}{\overline{M_1D}} \Leftrightarrow \overline{M_1R} = \sin\sphericalangle RDM_1 \cdot \overline{M_1D}$$

$$\Leftrightarrow \overline{M_1R} = \sin 65^\circ \cdot 4{,}07\text{ cm} = 3{,}69\text{ cm}$$

A 3.3

$$V_{ABC} = \frac{1}{3}\cdot r^2\cdot\pi\cdot h = \frac{1}{3}\cdot(5\text{ cm})^2\cdot\pi\cdot 10{,}72\text{ cm} = 280{,}65\text{ cm}^3$$

$$V_{EDC} = \frac{1}{3}\cdot r^2\cdot\pi\cdot h = \frac{1}{3}\cdot(4{,}07\text{ cm})^2\cdot\pi\cdot 8{,}72\text{ cm} = 151{,}26\text{ cm}^3$$

$$V_{ABDE} = 280{,}65\text{ cm}^3 - 151{,}26\text{ cm}^3 = 129{,}39\text{ cm}^3$$

$$V_{Halbkugel} = \frac{4}{6}\cdot r^3\cdot\pi = \frac{4}{6}\cdot(3{,}69\text{ cm})^3\cdot\pi = 105{,}23\text{ cm}^3$$

$$V_{gesamt} = 129{,}39\text{ cm}^3 + 105{,}23\text{ cm}^3 = 234{,}62\text{ cm}^3$$

A 3.4

$$O_{Halbkugel} = 2\cdot r^2\cdot\pi = 2\cdot(3{,}69\text{ cm})^2\cdot\pi = 85{,}55\text{ cm}^2$$

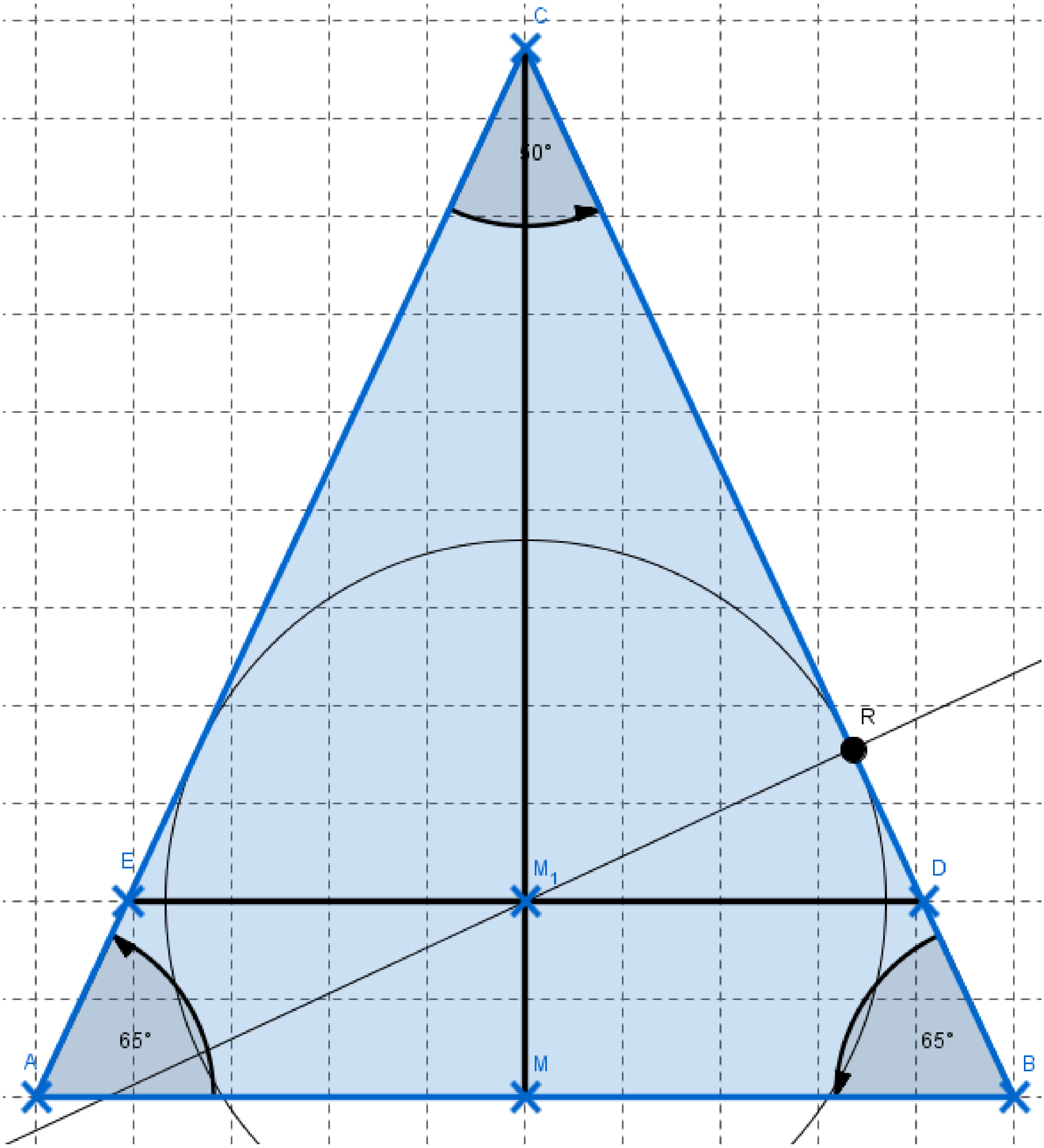
$$\sin\beta = \frac{h}{\overline{BD}} \Leftrightarrow \overline{BD} = \frac{h}{\sin\beta} = \frac{2\text{ cm}}{\sin 65^\circ} = 2{,}21\text{ cm}\ (= m)$$

$$M_{Kegelstumpf} = r_{unten} + r_{oben})\cdot\pi\cdot m = (5\text{ cm} + 3{,}69\text{ cm})\cdot\pi\cdot 2{,}21\text{ cm} = 60{,}33\text{ cm}^2$$

$$A_{Grundfläche} = r^2\cdot\pi = (5\text{ cm})^2\cdot\pi = 78{,}54\text{ cm}^2$$

$$A_{Deckfläche} = (r_{unten} - r_{oben})^2\cdot\pi = (1{,}31\text{ cm})^2\cdot\pi = 5{,}39\text{ cm}^2$$

$$O_{gesamt} = (85{,}55 + 60{,}33 + 78{,}54 + 5{,}39)\text{ cm}^2 = 229{,}81\text{ cm}^2$$


C
50°
R
E
M
1
D
65°
65°
A
M
B

Abschlussprüfung 1999
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Mathematik II — Aufgabengruppe B

Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 04.09.2013

Aufgabe B1

B 1.1 **g: $y = 0,5x + 2$**

p_1: $y = 0,25x^2 + bx + c$ $\quad$ R(0|2) T(10|7)

I $\quad 2 = 0 + 0 + c$

$\Leftrightarrow \quad c = 2$

II $\quad 7 = 0,25 \cdot (10^2) + 10b + c$

$\Leftrightarrow \quad 7 = 25 + 10b + 2$

$\Leftrightarrow \quad 10b = -20$

$\Leftrightarrow \quad b = -2$

Also: **p_1: $y = 0,25x^2 - 2x + 2$**

x	-1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0
y	4,25	2,00	0,25	-1,00	-1,75	-2,00	-1,75	-1,00	0,25	2,00	4,25	7,00

B 1.2

$A_1(1|2,5)$ $\quad$ $B_1(1|0,25)$ $\quad$ $C_1(3,68|3,84)$

$A_2(5,5|4,75)$ $\quad$ $B_2(5,5|-1,44)$ $\quad$ $C_2(8,19|6,09)$

$\tan \alpha' = \mathbf{0,5} \quad \Leftrightarrow \quad \alpha' = 26,66° \quad \Rightarrow \quad \alpha = 26,66° + 90° = 116,57°$

B 1.3

$$\overline{A_nB_n} = \sqrt{(x - x)^2 + (0,5x + 2 - 0,25x^2 + 2x - 2)^2}\ \text{cm}$$

$$\Leftrightarrow \overline{A_nB_n} = \sqrt{(-0,25x^2 + 2,5x)^2}\ \text{cm}$$

$$\Leftrightarrow \overline{A_nB_n} = (-0,25x^2 + 2,5x)\,\text{cm}$$

$$-0,25x^2 + 2,5x = 3$$

$$\Leftrightarrow -0,25x^2 + 2,5x - 3 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2,5 \pm \sqrt{2,5^2 - 4 \cdot (-0,25) \cdot (-3)}}{-0,5}$$

$$= \frac{-2,5 \pm \sqrt{3,25}}{-0,5} \Rightarrow x_1 = 1,39 \text{ und } x_2 = 8,61 \quad \Rightarrow \mathbb{L} = \{1,39;\ 8,61\}$$

$A_3(1,39|2,7)$ $\quad$ $A_4(8,61|6,31)$

B 1.4

$$\sin \sphericalangle C_nA_nF_n = \frac{\overline{C_nF_n}}{\overline{C_nA_n}} \Leftrightarrow \overline{C_nF_n} = \overline{C_nA_n} \cdot \sin \sphericalangle C_nA_nF_n$$

$$\Leftrightarrow \overline{C_nF_n} = 3\ \text{cm} \cdot \sin(180° - 116,57°) = 2,68\ \text{cm}$$

B 1.5

$\overline{A_nF_n}^2 = \overline{C_nA_n}^2 - \overline{C_nF_n}^2 = (3^2 - 2{,}68^2)\ cm^2 = 1{,}82\ cm^2$

$\Leftrightarrow \overline{A_nF_n} = 1{,}35$

$\overrightarrow{A_nC_n} = \begin{pmatrix} 2{,}68 \\ 1{,}35 \end{pmatrix}$

$\overrightarrow{A_nB_n} = \begin{pmatrix} 0 \\ 0{,}25x^2 - 2x + 2 - 0{,}5x - 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}25x^2 - 2{,}5x \end{pmatrix}$

$A(x) = 0{,}5 \cdot \begin{vmatrix} 0 & 2{,}68 \\ 0{,}25x^2 - 2{,}5x & 1{,}35 \end{vmatrix}$ FE

$\Leftrightarrow A(x) = [0{,}5 \cdot (-0{,}67x^2 + 6{,}7x)]$ FE

$\Leftrightarrow A(x) = [-0{,}335x^2 + 3{,}35x]$ FE

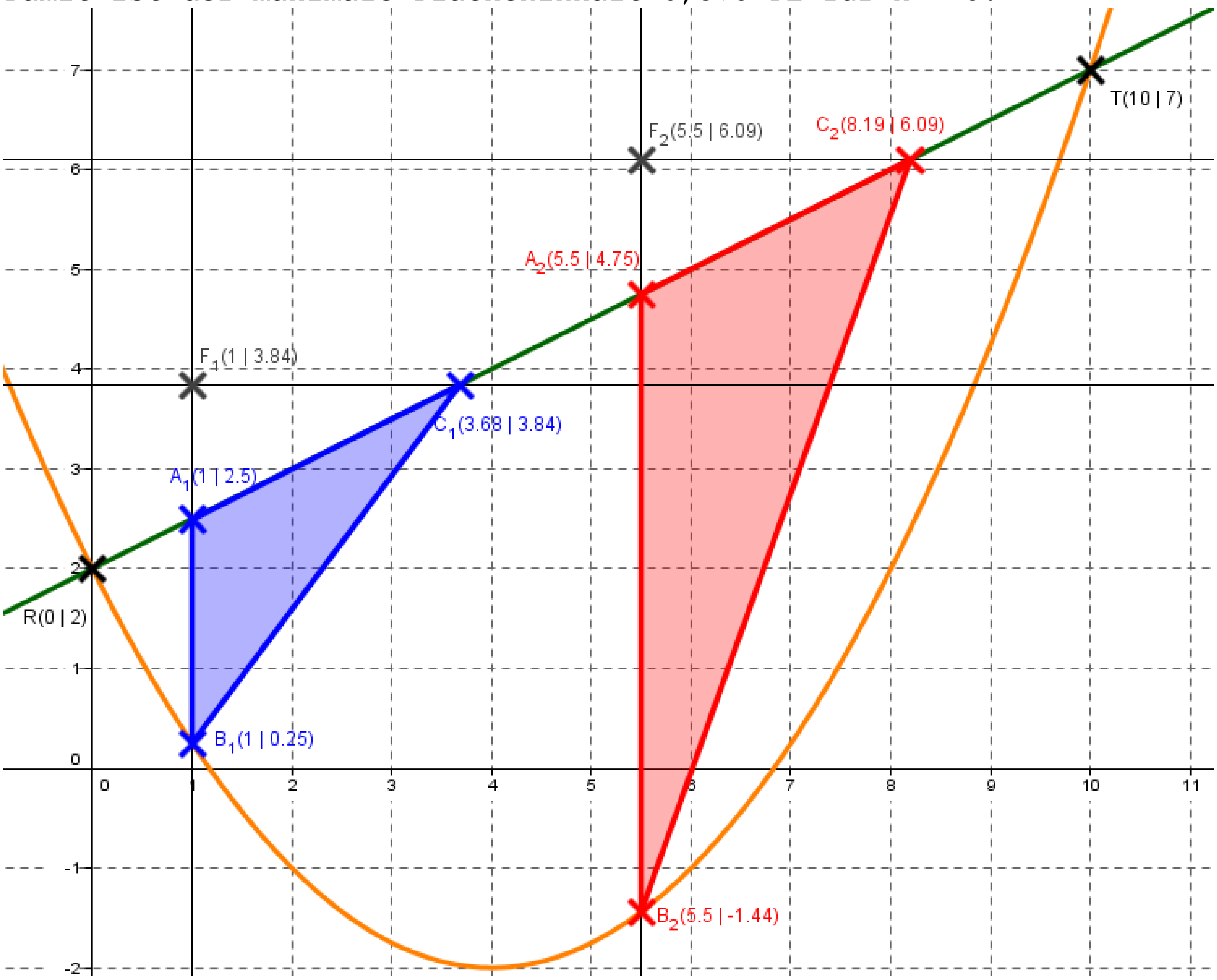
$A(x) = -0{,}335x^2 + 3{,}35x$

$\Leftrightarrow A(x) = -0{,}335(x^2 - 10x)$

$\Leftrightarrow A(x) = -0{,}335(x^2 - 10x + 5^2 - 5^2)$

$\Leftrightarrow A(x) = -0{,}335(x - 5)^2 + 8{,}375$

Damit ist der maximale Flächeninhalt 8,375 FE für x = 5.



Aufgabe B2

B 2.1

$$\overline{EA}^2 = \overline{AF}^2 + \overline{FE}^2 = (7{,}5^2 + 8^2)\ cm^2 = 120{,}25\ cm^2$$

$$\Leftrightarrow\ r = \overline{EA} = 10{,}97\ cm \quad \text{also } 11\ m$$

$$\tan \sphericalangle AEF = \frac{\overline{AF}}{\overline{FE}} = \frac{7{,}5\ cm}{8\ cm} = 0{,}94 \quad \Leftrightarrow\ \sphericalangle AEF = 43{,}2°$$

B 2.2

$$b = 2\cdot r\cdot \pi \cdot \frac{43{,}2° \cdot 2}{360°} = 2\cdot 11\ m\cdot \pi \cdot \frac{86{,}4°}{360°} = 16{,}6\ m$$

$$\sphericalangle DEA = 90° - 43{,}2° = 46{,}8°$$

$$\sphericalangle EAD = 180° - 46{,}8° - 100° = 33{,}2°$$

$$\frac{\overline{AD}}{\sin \sphericalangle DEA} = \frac{\overline{AE}}{\sin \sphericalangle EDA}$$

$$\Leftrightarrow\ \overline{AD} = \frac{\overline{AE}\cdot \sin \sphericalangle DEA}{\sin \sphericalangle EDA} = \frac{11\ m\cdot \sin 46{,}8°}{\sin 100°} = 8{,}1\ m$$

$$\frac{\overline{DE}}{\sin \sphericalangle EAD} = \frac{\overline{AE}}{\sin \sphericalangle EDA}$$

$$\Leftrightarrow\ \overline{DE} = \frac{\overline{AE}\cdot \sin \sphericalangle EAD}{\sin \sphericalangle EDA} = \frac{11\ m\cdot \sin 33{,}2°}{\sin 100°} = 6{,}1\ m$$

$$u_{gesamt} = 6{,}1\ m + 6{,}1\ m + 8{,}1\ m + 8{,}1\ m + 16{,}6\ m = 45\ m$$

B 2.3

$$\sphericalangle EMR = 180° - 46{,}8° - 46{,}8° = 86{,}4°$$

$$\overline{BD}^2 = \overline{BC}^2 + \overline{CD}^2 - 2\cdot \overline{BC}\cdot \overline{CD}\cdot \cos \sphericalangle DCB$$

$$\Leftrightarrow\ \overline{BD}^2 = (8{,}2^2 + 12{,}2^2 - 2\cdot 8{,}2\cdot 12{,}2\cdot \cos 100°)\ m^2 = 250{,}8\ m^2$$

$$\Leftrightarrow\ \overline{BD} = 15{,}8\ m$$

$$\frac{\overline{CB}}{\sin \sphericalangle BDE} = \frac{\overline{BD}}{\sin \sphericalangle DCB}$$

$$\Leftrightarrow\ \sin \sphericalangle BDE = \frac{\overline{CB}\cdot \sin \sphericalangle DCB}{\overline{BD}} = \frac{8{,}1\ m\cdot \sin 100°}{15{,}8\ m} = 0{,}5$$

$$\Leftrightarrow\ \sphericalangle BDE = 30{,}3° \quad (149{,}7° \text{ nicht möglich wegen } \sphericalangle ADE = 100°)$$

$$\frac{\overline{ME}}{\sin \sphericalangle BDE} = \frac{\overline{DE}}{\sin \sphericalangle EMD} \Leftrightarrow\ \overline{ME} = \frac{\overline{DE}\cdot \sin \sphericalangle BDE}{\sin \sphericalangle EMD}$$

$$\Leftrightarrow\ \overline{ME} = \frac{6{,}1\ m\cdot \sin 30{,}3°}{\sin(180° - 30{,}3° - 46{,}8°)} = 3{,}2\ m$$

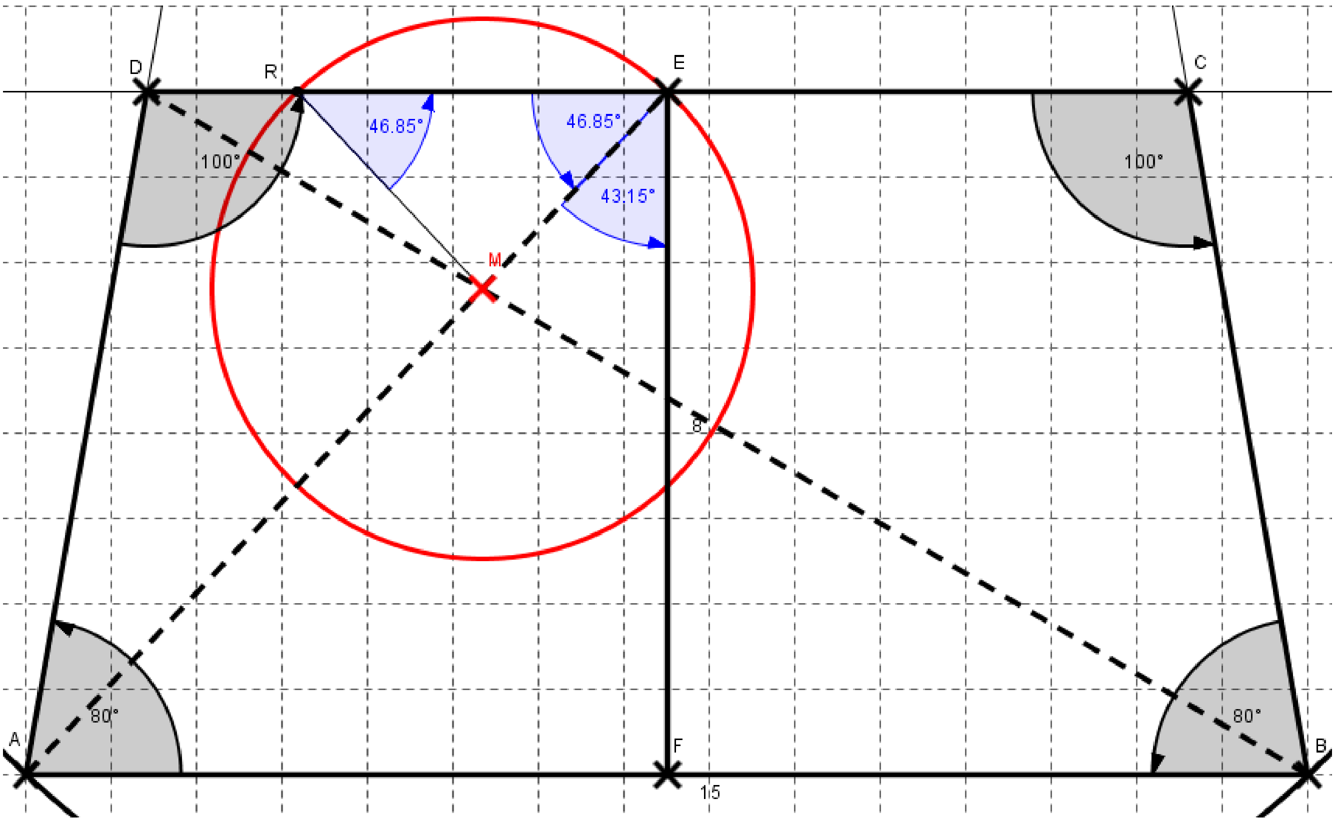
B 2.4

$$A_{Sektor} = r^2 \cdot \pi \cdot \frac{\sphericalangle EMR}{360°}$$

$$\Leftrightarrow A_{Sektor} = (3{,}2\ m)^2 \cdot \pi \cdot \frac{86{,}4°}{360°} = 7{,}7\ m^2$$

$A_{MER} = 0{,}5 \cdot r \cdot r \cdot \sin 86{,}4° = 0{,}5 \cdot (3{,}2\ m)^2 \cdot \sin 86{,}4° = 5{,}1\ m^2$

$A_{außen} = 7{,}7\ m^2 - 5{,}1\ m^2 = 2{,}6\ m^2$



Aufgabe B3

B 3.1

$$\tan\gamma = \frac{\overline{AS}}{\overline{AC}} = \frac{9\ \text{cm}}{10\ \text{cm}} = 0{,}9 \quad \Leftrightarrow \gamma = 41{,}99°$$

$$\overline{CS}^2 = \overline{AS}^2 + \overline{AC}^2 = (9^2 + 10^2)\ \text{cm}^2 = 181\ \text{cm}^2$$

$$\Leftrightarrow \overline{CS} = 13{,}45\ \text{cm}$$

B 3.2

$\sphericalangle MP_1C = 180° - 70° - 41{,}99° = 68{,}01°$

$$\frac{\overline{CP_1}}{\sin \sphericalangle CMP_1} = \frac{\overline{MC}}{\sin \sphericalangle MP_1C}$$

$$\Leftrightarrow \overline{CP_1} = \frac{\overline{MC}\cdot\sin \sphericalangle CMP_1}{\sin \sphericalangle MP_1C} = \frac{7\ \text{cm}\cdot\sin 70°}{\sin 68{,}01°} = 7{,}09\ \text{cm}$$

B 3.3

$$\sin\gamma = \frac{h}{\overline{CP_1}} \Leftrightarrow h = \sin\gamma \cdot \overline{CP_1} = \sin 41{,}99° \cdot 7{,}09\ \text{cm} = 4{,}74\ \text{cm}$$

$$V = \frac{1}{3} \cdot A_G \cdot h = \frac{1}{3} \cdot 0{,}5 \cdot 7\ \text{cm} \cdot 4\ \text{cm} \cdot 4{,}74\ \text{cm} = 22{,}12\ \text{cm}^2$$

B 3.4

$A_{ABCD} = 0{,}5 \cdot e \cdot f = 0{,}5 \cdot 10\ \text{cm} \cdot 8\ \text{cm} = 40\ \text{cm}^2$

$A_{BDP2} = 40\ \text{cm}^2 \cdot 0{,}8 = 32\ \text{cm}^2$

$$32\ \text{cm}^2 = 0{,}5 \cdot \overline{BD} \cdot \overline{MP_2}$$

$$\Leftrightarrow \overline{MP_2} = \frac{32\ \text{cm}^2}{0{,}5 \cdot \overline{BD}} = \frac{32\ \text{cm}^2}{0{,}5 \cdot 8\ \text{cm}} = 8\ \text{cm}$$

$$\frac{\overline{MP_2}}{\sin\gamma} = \frac{\overline{MC}}{\sin \sphericalangle MP_2C}$$

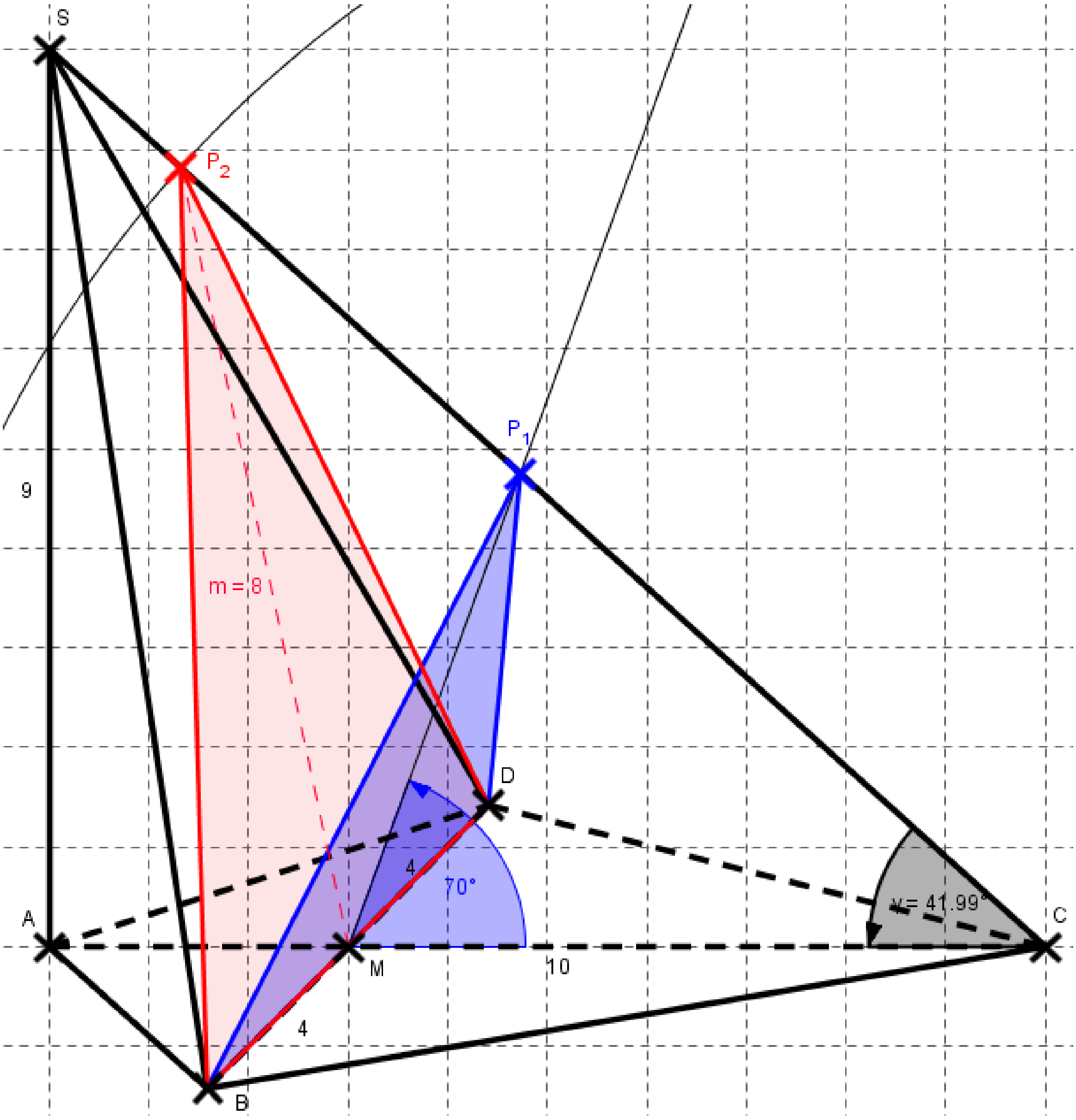
$$\Leftrightarrow \sin \sphericalangle MP_2C = \frac{\overline{MC}\cdot\sin\gamma}{\overline{MP_2}} = \frac{7\ \text{cm}\cdot\sin 41{,}99°}{8\ \text{cm}} = 0{,}59$$

$\Leftrightarrow \sphericalangle MP_2C = 35{,}83°$ (144,17° wegen $\gamma = 41{,}99°$ (Innenwinkelsumme) nicht möglich)

$$\frac{\overline{MP_2}}{\sin\gamma} = \frac{x}{\sin \sphericalangle CMP_2} \Leftrightarrow x = \frac{\overline{MP_2}\cdot\sin \sphericalangle CMP_2}{\sin\gamma}$$

$$\Leftrightarrow x = \frac{8\ \text{cm}\cdot\sin(180° - 41{,}99° - 35{,}83°)}{\sin 41{,}99°} = 11{,}69\ \text{cm}$$

[Zeichnung nur zur Kontrolle]


S
P2
P1
9
m = 8
D
4
70°
A
M
10
γ= 41.99°
C
4
B