Abschlussprüfung 1991
an den Realschulen in Bayern

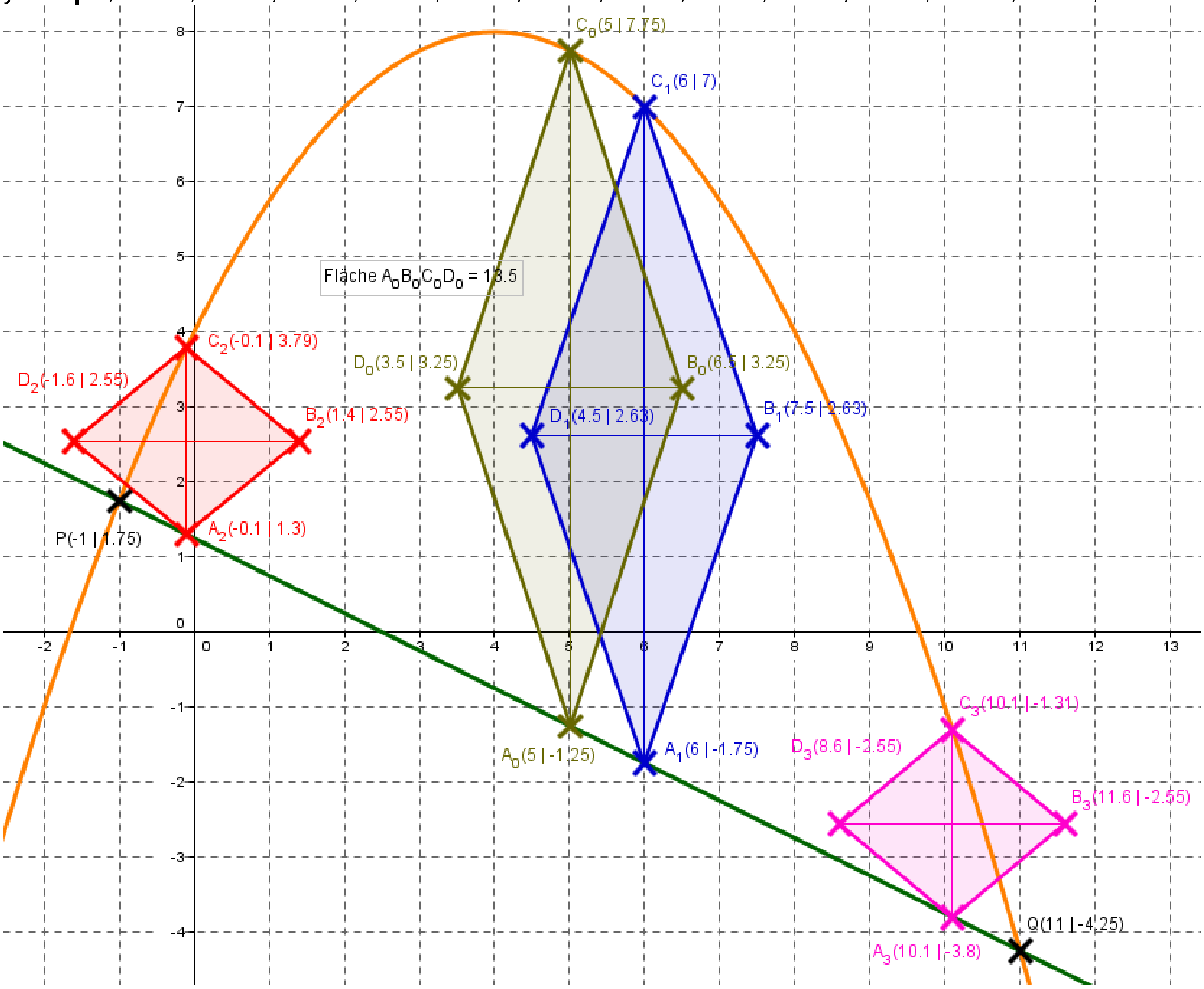
Mathematik II — Aufgabengruppe B

Lösungsvorschlag von StR(RS) Karsten Reibold – Stand: 28.07.2013

Aufgabe B1 **f_1: $y = -0{,}25x^2 + 2x + 4$** **f_2: $y = -0{,}5x + 1{,}25$**

B 1.1

x	-1,0	0,0	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0	11,0
y	1,75	4,00	5,75	7,00	7,75	8,00	7,75	7,00	5,75	4,00	1,75	-1,00	-4,25



B 1.2

$$-0{,}25x^2 + 2x + 4 = -0{,}5x + 1{,}25$$

$$\Leftrightarrow -0{,}25x^2 + 2{,}5x + 2{,}75 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2{,}5 \pm \sqrt{2{,}5^2 - 4\cdot(-0{,}25)\cdot 2{,}75}}{-0{,}5}$$

$$= \frac{-2{,}5 \pm \sqrt{9}}{-0{,}5} \Rightarrow x_1 = -1 \text{ und } x_2 = 11 \quad \mathbb{L} = \{-1;\ 11\}$$

$$\Rightarrow P(-1 \mid 1{,}75) \quad Q(11 \mid -4{,}25)$$

B 1.3 **$A_1(6|-1,75)$, $B_1(7,5|2,63)$, $C_1(6|7)$, $D_1(4,5|2,63)$**

B 1.4

$\overline{A_nC_n} = \sqrt{[0]^2 + [-0,25x^2 + 2x + 4 - (-0,5x + 1,25)]^2}$ LE

$\Leftrightarrow \overline{A_nC_n} = (-0,25x^2 + 2,5x + 2,75)$ LE

$A(x) = 0,5 \cdot \overline{B_nD_n} \cdot \overline{A_nC_n}$

$\Leftrightarrow A(x) = 0,5 \cdot 3 \cdot (-0,25x^2 + 2,5x + 2,75)$ FE

$\Leftrightarrow A(x) = (-0,375x^2 + 3,75x + 4,125)$ FE

B 1.5 $A(x) = -0,375(x^2 - 10x + 5^2 - 5^2) + 4,125$

$\Leftrightarrow A(x) = -0,375(x - 5)^2 + 13,5$

Für **$x = 5$** wird der Flächeninhalt mit 13,5 FE maximal.

B 1.6 Ein Quadrat hat gleich lange Diagonalen $\Rightarrow \overline{A_nC_n} = 3$ LE

$-0,25x^2 + 2,5x + 2,75 = 3$

$\Leftrightarrow -0,25x^2 + 2,5x - 0,25 = 0$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2,5 \pm \sqrt{2,5^2 - 4 \cdot (-0,25) \cdot 0,25}}{-0,5}$$

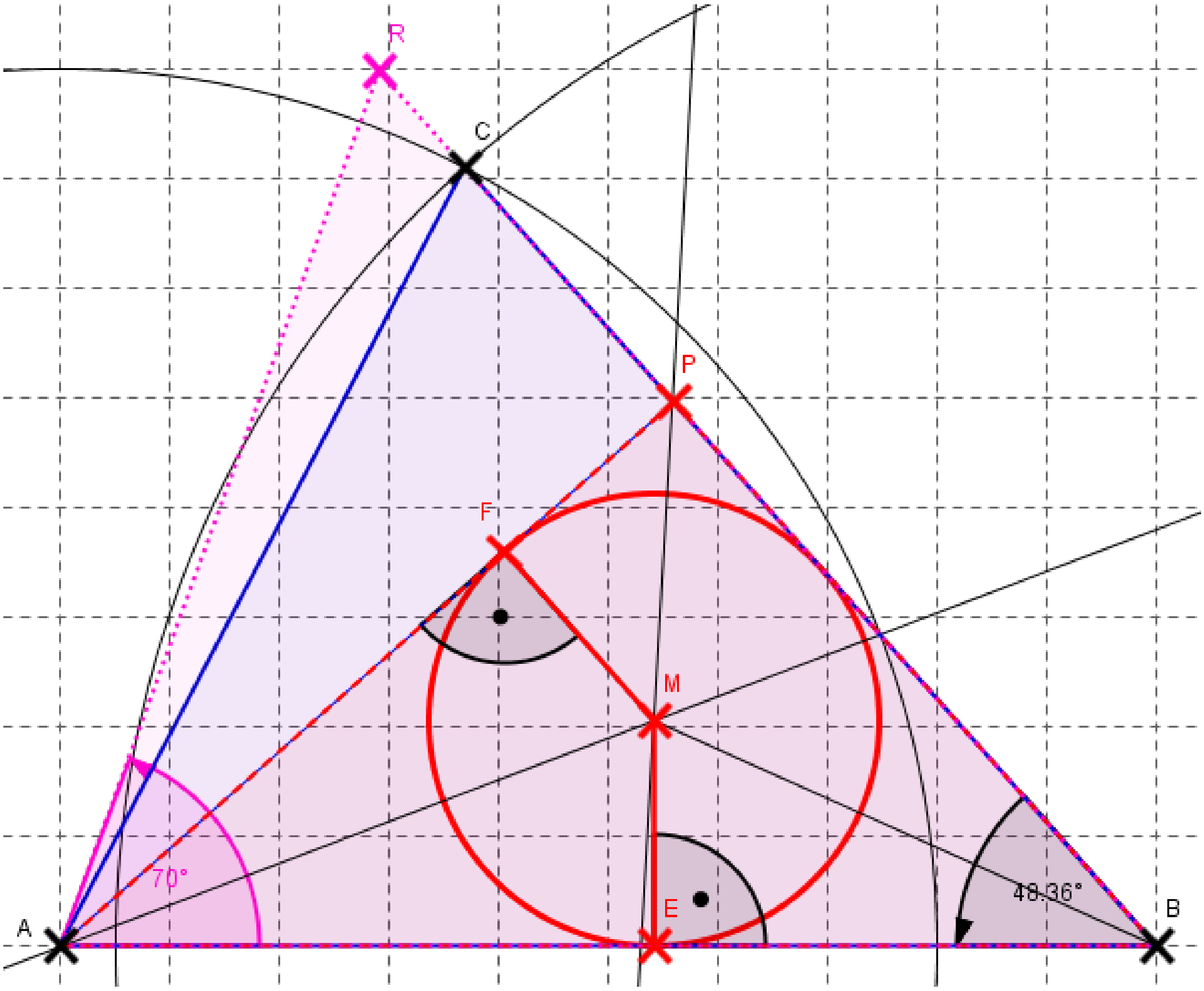
$$= \frac{-2,5 \pm \sqrt{6,5}}{-0,5} \Rightarrow x_1 = -0,1 \text{ und } x_2 = 10,1 \quad \mathbb{L} = \{-0,1;\ 10,1\}$$

$\Rightarrow$ **$A_2(-0,1\ |1,2)$** und **$A_3(10,1\ |-3,8)$**

[Zeichnung ergänzt mit allen berechneten Rauten / Quadraten]

Aufgabe B2

B 2.1



$$\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2 - 2 \cdot \overline{AB} \cdot \overline{BC} \cdot \cos \sphericalangle CBA$$

$$\Leftrightarrow \cos \sphericalangle CBA = \frac{\overline{AC}^2 - \overline{AB}^2 - \overline{BC}^2}{-2 \cdot \overline{AB} \cdot \overline{BC}} = \frac{8^2 - 10^2 - 9{,}5^2}{-2 \cdot 10 \cdot 9{,}5} = 0{,}66$$

$$\Leftrightarrow \sphericalangle CBA = 48{,}36°$$

$$\sin \sphericalangle CBA = \frac{\overline{AP}}{\overline{AB}} \Leftrightarrow \overline{AP} = \sin \sphericalangle CBA \cdot \overline{AB} = \sin 48{,}36° \cdot 10 \text{ cm} = 7{,}47 \text{ cm}$$

B 2.2 Inkreismittelpunkt: Schnittpunkt der Winkelhalbierenden.

B 2.3

$\sphericalangle BAP = 180° - 90° - 48{,}36° = 41{,}64°$

$A_{APB} = 0{,}5 \cdot 7{,}47\ cm \cdot 10\ cm \cdot \sin 41{,}64° = 24{,}82\ cm^2$

$\cos \sphericalangle CBA = \frac{\overline{BP}}{\overline{AB}} \Leftrightarrow \overline{BP} = \cos \sphericalangle CBA \cdot \overline{AB} = \cos 48{,}36° \cdot 10\ cm = 6{,}64\ cm$

$\overline{ME} = r_{Inkreis} = \frac{2 \cdot A_{APB}}{a + b + c} = \frac{2 \cdot 24{,}82\ cm^2}{10\ cm + 7{,}47\ cm + 6{,}64\ cm} = 2{,}06\ cm$

$\sphericalangle BAM = \sphericalangle BAP : 2 = 20{,}82°$

$\tan \sphericalangle BAM = \frac{\overline{ME}}{\overline{AE}} \Leftrightarrow \overline{AE} = \frac{\overline{ME}}{\tan \sphericalangle BAM} = \frac{2{,}06\ cm}{\tan 20{,}82°} = 5{,}42\ cm$

$\Rightarrow \overline{EB} = \overline{AB} - \overline{AE} = 10\ cm - 5{,}42\ cm = 4{,}58\ cm$

$\tan \sphericalangle MBE = \frac{\overline{ME}}{\overline{EB}} = \frac{2{,}06\ cm}{4{,}58\ cm} = 0{,}45 \Leftrightarrow \sphericalangle MBE = 24{,}22°$

$\sphericalangle AMB = 180° - \sphericalangle BAM - \sphericalangle MBE = 180° - 20{,}82° - 24{,}22° = 143{,}96°$

B 2.4

$\overline{AM}^2 = \overline{AE}^2 + \overline{ME}^2 = (5{,}42\ cm)^2 + (2{,}06\ cm)^2 = 33{,}62\ cm^2$

$\Leftrightarrow \overline{AM} = 5{,}8\ cm$

$A_{AEMF} = 2 \cdot 0{,}5 \cdot \overline{AM} \cdot \overline{AE} \cdot \sin \sphericalangle BAM$

$\Leftrightarrow A_{AEMF} = 5{,}8\ cm \cdot 5{,}42\ cm \cdot \sin 20{,}82° = 11{,}17\ cm^2$

$\sphericalangle FME = 360° - 41{,}64° - 180° = 138{,}36°$

$A_{Kreissektor} = r^2 \cdot \pi \cdot \frac{138{,}36°}{360°} = 4{,}2436\ cm^2 \cdot \pi \cdot \frac{138{,}36°}{360°} = 5{,}12\ cm^2$

$A_{konkaveFläche} = 11{,}17\ cm^2 - 5{,}12\ cm^2 = 6{,}05\ cm^2$

B 2.5

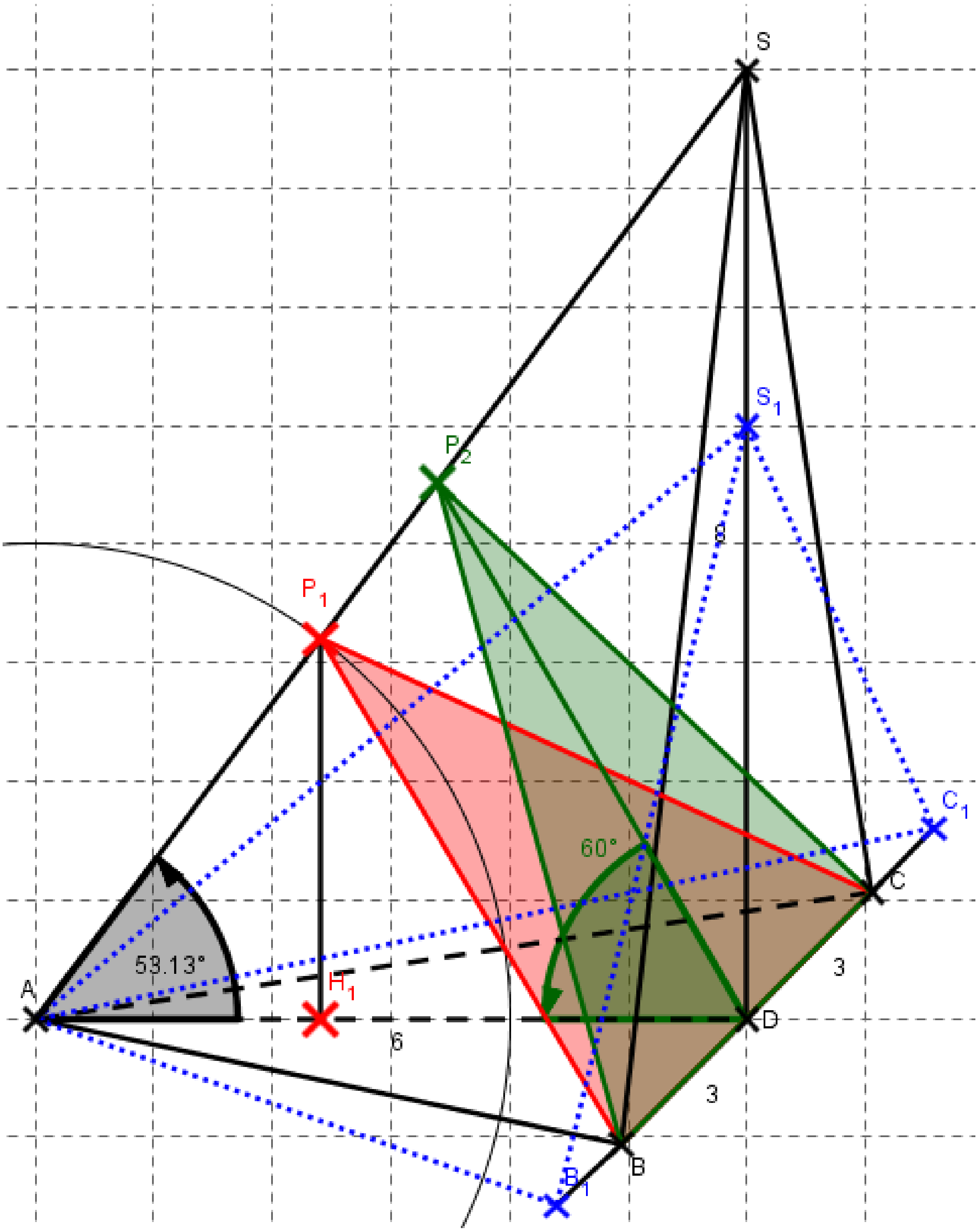
$\sphericalangle ARB = 180° - 70° - 48{,}36° = 61{,}64°$

$\frac{\overline{AB}}{\sin \sphericalangle ARB} = \frac{\overline{BR}}{\sin \sphericalangle BAR} \Leftrightarrow \overline{BR} = \frac{\overline{AB} \cdot \sin \sphericalangle BAR}{\sin \sphericalangle ARB} = \frac{10\ cm \cdot \sin 70°}{\sin 61{,}64°}$

$\Leftrightarrow \overline{BR} = 10{,}68\ cm \Rightarrow \overline{CR} = \overline{BR} - \overline{BC} = 10{,}68\ cm - 9{,}5\ cm = 1{,}18\ cm$

Aufgabe B3

B 3.1



$$\tan \sphericalangle DAS = \frac{\overline{SD}}{\overline{AD}} = \frac{8\ cm}{6\ cm} = 1\frac{1}{3} \Leftrightarrow \sphericalangle DAS = 53{,}13°$$

B 3.2

$$\sin \sphericalangle DAS = \frac{\overline{P_1H_1}}{\overline{AP_1}} \Leftrightarrow \overline{P_1H_1} = \sin \sphericalangle DAS \cdot \overline{AP_1}$$

$\Leftrightarrow \overline{P_1H_1} = \sin 53{,}13° \cdot 4\ cm = 3{,}2\ cm$

$\overline{AH_1}^2 = \overline{AP_1}^2 - \overline{P_1H_1}^2 = (4\ cm)^2 - (3{,}2\ cm)^2 = 5{,}76\ cm^2$

$\Leftrightarrow \overline{AH_1} = 2{,}4\ cm \quad \Rightarrow \overline{H_1D} = 6\ cm - 2{,}4\ cm = 4{,}6\ cm$

$\overline{P_1D}^2 = \overline{P_1H_1}^2 + \overline{H_1D}^2 = (3{,}2\ cm)^2 + (4{,}6\ cm)^2 = 31{,}4\ cm^3$

$\Leftrightarrow \overline{P_1D} = 5{,}6\ cm$

$A_{BCP1} = 0{,}5 \cdot \overline{BC} \cdot \overline{P_1D} = 0{,}5 \cdot 6\ cm \cdot 5{,}6\ cm = 16{,}8\ cm^2$

B 3.3

$\sphericalangle AP_2D = 180° - 60° - 53{,}13° = 66{,}87°$

$$\frac{\overline{P_2D}}{\sin \sphericalangle DAS} = \frac{\overline{AD}}{\sin \sphericalangle AP_2D} \Leftrightarrow \overline{P_2D} = \frac{\overline{AD} \cdot \sin \sphericalangle DAS}{\sin \sphericalangle AP_2D}$$

$$\Leftrightarrow \overline{P_2D} = \frac{6\ cm \cdot \sin 53{,}13°}{\sin 66{,}87°} = 5{,}22\ cm$$

$$\tan \sphericalangle BP_2C = \frac{\overline{BD}}{\overline{P_2D}} = \frac{3\ cm}{5{,}22\ cm} = 0{,}57 \Leftrightarrow \sphericalangle BP_2C = 29{,}89°$$

B 3.4

$$V(a) = \frac{1}{3} \cdot A_G \cdot h = [\frac{1}{3} \cdot 0{,}5 \cdot (6 + 2a) \cdot 6 \cdot (8 - 2a)]cm^3$$

$\Leftrightarrow \quad V(a) = (6 + 2a)(8 - 2a)cm^3 = (48 - 12a + 16a - 4a^2)\ cm^3$

$\Leftrightarrow \quad V(a) = [4(-a^2 + a + 12)]cm^3$

B 3.5

$$\tan 50° = \frac{\overline{DC_2}}{\overline{S_2D}} \Leftrightarrow \tan 50° = \frac{3 + x}{8 - 2x}$$

$\Leftrightarrow \quad \tan 50° (8 - 2x) = 3 + x$

$\Leftrightarrow \quad 9{,}53 - 2{,}38x = 3 + x$

$\Leftrightarrow \quad 6{,}53 = 3{,}38x \quad \Leftrightarrow x = 1{,}93$

$V(1{,}93) = [4(-1{,}93^2 + 1{,}93 + 12)]cm^3 = 40{,}82\ cm^3$